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DEPARTMENT OF CIVIL ENGINEERING

REGULATION 2023

U23CET41 – STRNGTH OF MATERIALS II

SUBJECT NOTES

Strength of Materials - II [U22CET412]

L	T	P	C
3	0	0	3

UNIT - I Compression Member

Column - Type - Mode of failure - Buckling Load - Factor of Safety - Euler's Theory - Different end condition - Rankine's - Gordon formula - Axial & eccentric loads - Direct, bending & combined bending stress - Calculation of Combined bending stress - core Section - Middle third & Middle fourth Rule.

Column/Strut:

It is defined as member of Structure which is subjected to axial compressive load.

If member of structure is vertical and both of its ends are fixed rigidly while subjected to axial compressive load.

Ex: Vertical Pillar b/w floor and roof.

If member of structure is not vertical and one or both of its end are hinged or rigid, then bar is known as Strut.

Ex: Connecting rod, Piston rods etc.

Definition:

Column : A Column is long Vertical Slender bar or vertical member subjected to axial compressive load and fixed rigidly at both ends.

Strut : A Strut is slender bar in any position other than vertical subjected to compressive load and fixed rigidly or hinged or pin jointed at one or both the ends.

Slenderness ratio (k) : It is ratio of unsupported length to min radius of gyration of c/s end of column. It has no unit

$$k = \frac{\text{unsupported length}}{\text{min. radius of gyration}} \quad \left(\frac{l_e}{k_i}\right).$$

Buckling factor: It is ratio b/w equivalent length of colum to min radius of gyration.

$$= \frac{\text{Equivalent length of columm}}{\text{min. radius of gyration}}$$

Buckling load: The max limiting load at which column tend to have lateral displacement or tends to buckle.

* Buckling Take place about axis having min radius of gyration or least M.O.I.

Safe load: It is load to which a column is actually subjected to and is well below buckling load.

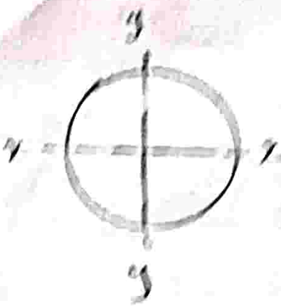
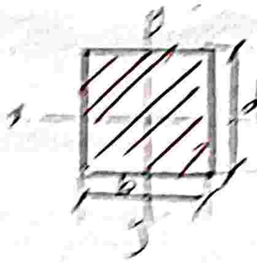
$$\text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of Safety}}$$

Column [K cos l to D].

Short column	Intermediate column	Long column
[$L < 8d$ $K < 32$]	[$L = 8d$ to $30d$ $K = 32$ to 120]	[$L \geq 30d$ $K > 120$]
* Buckling is neglible * direct comp. Stress. is more	* Both buckling cy direct stress occur	* direct comp stress neglible * Buckling stress is more.

a Long column: It is that column in which eff. length to Least lateral dimension ratio is greater than 12

b Short column: It is that column in which ratio of eff. length to L.L.D less than 12.

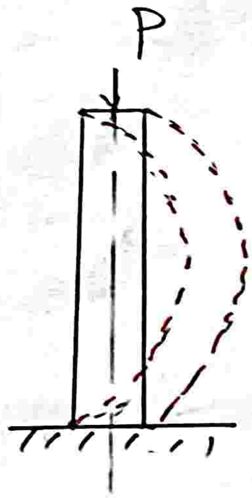
	
For long column: $\left(\frac{L_e}{D} > 12\right)$	For long column $\left(\frac{L_e}{b} > 12\right)$
For short column: $\left(\frac{L_e}{D} < 12\right)$	For short column $\left(\frac{L_e}{b} < 12\right)$

* If the column is long then, it will fail only because of buckling (bending).
* If the column is short then, it will fail only because of crushing (direct compression).
* For long column, Euler's theory is used.
* For short column, Rankine's theory is used.

Euler's Theory:

In 1757, Swiss mathamatician Leonhard Euler gave the formula for stability of long column.

Euler Considering only bending of column as direct compression was negligible compared to bending.



Hence Euler's Theory is applicable only for long column.

Assumption in Euler's theory:

Column is subjected to axial loading ey Perfectly st.

Material of column is homogenous ey isotropic
(Same material) (Same E).

Material of column is elastic ey obeys hooke's law
(σ vs e)

c/s of column is uniform throughout its length

Length of column is very large as compared to lateral dimension.

Self weight of column is neglible.

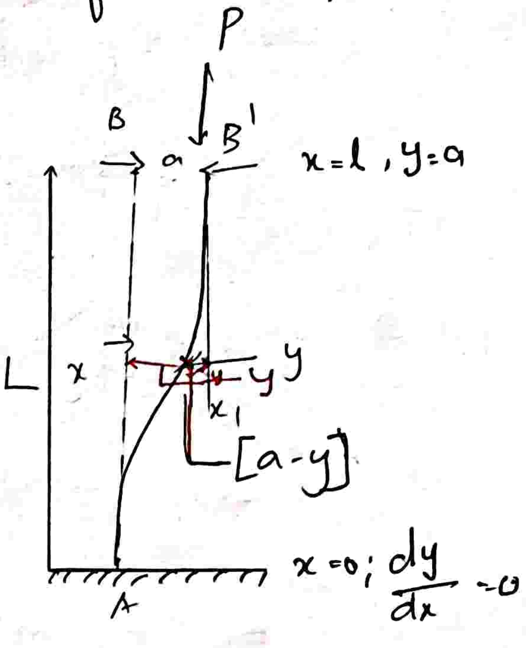
The column is straight before loading

Column will fail by buckling alone.

End condition for long column:

* Both the end of column are hinged (or Pinned)
* One end is fixed & other end is free.
* Both the end of column are fixed
* One end is fixed & other is pinned.

Expression for Crippling load when One end is fixed & other end is free.



Taking moment about A ; Point

$$M = P(a-y)$$
$$= Pa - Py \rightarrow (1)$$

deflection eq ①

$$EI \frac{d^2y}{dx^2} = Pa - Py$$

$$EI \frac{d^2y}{dx^2} + Py = Pa$$

$$\frac{d^2y}{dx^2} + \frac{P \cdot y}{EI} = \frac{P}{EI} a \rightarrow ②$$

General Soln of This eqn:

$$y = A \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] + \underbrace{B \sin\left[x \cdot \sqrt{\frac{P}{EI}}\right]}_{(C.F)} + \underbrace{a}_{(P.I)}$$

①

$$\frac{dy}{dx} = -A \sin\left[x \cdot \sqrt{\frac{P}{EI}}\right] \sqrt{\frac{P}{EI}} + B \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] + \sqrt{\frac{P}{EI}} + 0$$

$$\boxed{\frac{dy}{dx} = 0}, \quad \boxed{x = 0}$$

$$0 = B \cdot \sqrt{\frac{P}{EI}} \Rightarrow B = 0$$

$$y = A \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] + B \sin\left[x \cdot \sqrt{\frac{P}{EI}}\right] + a.$$

$x = 0$, $y = 0$

$$0 = A + 0 + a$$

$$\boxed{A = -a} \quad \boxed{B = 0}$$

$$y = -a \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] + a$$

$$y = a\left[1 - \cos\left\{x \cdot \sqrt{\frac{P}{EI}}\right\}\right]$$

$x = L$; $y = a$

$$a = a\left[1 - \cos\left[L \cdot \sqrt{\frac{P}{EI}}\right]\right]$$

$$1 = 1 - \cos\left\{L \cdot \sqrt{\frac{P}{EI}}\right\}$$

$$\cos\left(L \cdot \sqrt{\frac{P}{EI}}\right) = 0$$

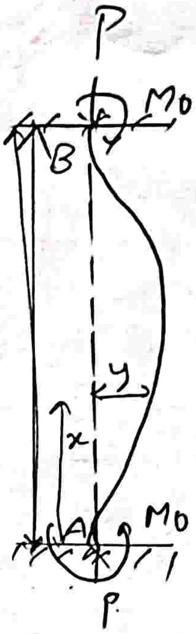
$$L\sqrt{\frac{P}{EI}} = \frac{\pi}{2} \qquad \left[\cos \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} = 0\right]$$

$$\sqrt{\left(\frac{P}{EI}\right)} = \frac{\pi}{2L}$$

$$\boxed{P = \frac{\pi^2}{4L^2} \cdot EI}$$

One end fixed & other end free.

Column with both ends are fixed:



M_0 = ~~fixed~~ end moment at A & B.

Moment at the Section = $M_0 - P.y$.

Moment at section also = $EI.\dfrac{d^2y}{dx^2}$

$$EI.\frac{d^2y}{dx^2} = M_0 - Py$$

$$EI.\frac{d^2y}{dx^2} + P.y = M_0$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_0}{E.I} \Rightarrow (1)$$

$$y = A\cos\left[x\sqrt{\frac{P}{EI}}\right] + B\sin\left[x\sqrt{\frac{P}{EI}}\right] + \frac{M_0}{P} \quad \rightarrow (2)$$

At $x = 0$ $y = 0$

$$0 = A\cos(0) + B\sin 0 + \frac{M_0}{P}.$$

$$\boxed{A = \frac{-M_0}{P}} \Rightarrow (3)$$

diff. eqn (2)

$$\frac{dy}{dx} = -A\cdot\sin\left[x\sqrt{\frac{P}{EI}}\right]\sqrt{\frac{P}{EI}} + B\cos\left[x\sqrt{\frac{P}{EI}}\right]\sqrt{\frac{P}{EI}} + 0$$

$$\frac{dy}{dx} = -A\sqrt{\frac{P}{EI}}\sin\left(x\sqrt{\frac{P}{EI}}\right) + B\sqrt{\frac{P}{EI}}\cos\left[x\sqrt{\frac{P}{EI}}\right] \quad \rightarrow (4).$$

At $x = 0$ $\frac{dy}{dx} = 0$

$$0 = B\sqrt{\frac{P}{EI}}$$

$$\boxed{B = 0}$$

Sub $B=0$ in Eqn ③.

$$y = \frac{-M_0}{P} \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] + \frac{M_0}{P} \Rightarrow ⑥$$

if $x = l$ $y = 0$.

$$0 = \frac{-M_0}{P} \cos\left[l \cdot \sqrt{\frac{P}{EI}}\right] + \frac{M_0}{P}.$$

$$0 = \frac{M_0}{P}\left[1 - \cos\left[l \sqrt{\frac{P}{EI}}\right]\right]$$

$$\frac{M_0}{P} \cos\left[l \cdot \sqrt{\frac{P}{EI}}\right] = \frac{M_0}{P}$$

$$\cos\left[l \cdot \sqrt{\frac{P}{EI}}\right] = \frac{M_0}{P} \times \frac{P}{M_0}$$

$$\cos\left[l \sqrt{\frac{P}{EI}}\right] = 1$$

$[\cos 0, \cos 2\pi, \cos 4\pi = 1]$

$$l\sqrt{\frac{P}{EI}} = 2\pi.$$

$$\sqrt{\frac{P}{EI}} = \frac{2\pi}{l} \Rightarrow \frac{P}{EI} = \frac{4\pi^2}{l^2}$$

$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$ Both ends are fixed

S.no	End conditions of Column	Gripptlug load in terms of Actual length	Gripptlug load in terms of effective length	Reltion b/w eff. length & actual length
1	Both end hinged	$\frac{\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$L_e = l$
2	One end is free & other is fixed	$\frac{\pi^2 EI}{4l^2}$	$\frac{\pi^2 EI}{L_e^2}$	$L_e = 2l$
3	Both ends are fixed	$\frac{4\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$L_e = \frac{l}{2}$
4	One end fixed & other is hinged.	$\frac{2\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$L_e = \frac{l}{\sqrt{2}}$

P-1 A S.S.B of length 4m is subjected to UDL of 30 kN/m over the whole span & deflects 15mm at the centre. Determine the Crippling load when this beam is used as column with following conditions:

i) one end fixed & other end hinged

ii) both end pin jointed.

$l = 4m \Rightarrow 4000mm$

UDL, $w = 30\ kN/m \Rightarrow 30000\ N/m$

$\Rightarrow \frac{30000}{1000} \frac{N}{mm}$

$\Rightarrow 30\ N/mm$

deflection @ Centre (y) = 15mm.

$$y = \frac{5}{384} \times \frac{wL^4}{EI}$$

$$15 = \frac{5}{384} \times \frac{30 \times (4000)^4}{EI}$$

$$E.I = \frac{5}{384} \times \frac{30(4000)^4}{15}$$

$$= \frac{5}{384} \times \frac{3 \times 256}{15} \times 10^{13}$$

$$= \cancel{2}.0130 \times 5.12 \times 10^{14}$$

$$E.I \Rightarrow 6.656 \times 10^{12} \text{ N.mm}^2$$

(i) Column with One end fixed & other hinged:

$$P = \frac{2\pi^2 \times EI}{l^2}$$

$$= \frac{2 \times \pi^2 \times 6.656 \times 10^{12}}{(4000)^2}$$

$$= 8.2 \times 10^6 \text{ N}$$

$1 KN = 1000 N$

$$P = 8203 \text{ KN}$$

$1 N = \frac{1}{1000} KN.$

(ii) Both ends are Pin jointed:

$$P = \frac{\pi^2 \times EI}{l^2}$$

$$= \frac{\pi^2 \times 6.656 \times 10^{12}}{(4000)^2}$$

$$= 4.097 \times 10^6 \text{ N}$$

$$P = 4097 \text{ kN}.$$

P-2 For what length of mild steel bar 50mm dia used as column, the euler's theory is applicable if the ultimate comp strength is 350 N/mm² & E = 210 GPa when both end are fixed

G.D $d = 50\text{mm}$.

Rankine constant $\sigma_c = 350 \text{ N/mm}^2$

$E = 210 \text{ kN/mm}^2 = 210 \times 10^3 \text{ N/mm}^2$.

$$P_E = \frac{\pi^2 EI}{l_e^2}$$

$$I = \frac{\pi}{64}(d)^4 = \frac{\pi}{64}(50)^4$$

$$I = 306.8 \times 10^3 \text{ mm}^4.$$

$$\sigma_c = \frac{P_E}{A} = \frac{\pi^2 EI}{A l_e^2}$$

$$\frac{\pi^2 EI}{A\, l_e^2} = 350$$

$$l_e^2 = \frac{\pi^2 EI}{A \times 350}$$

$$= \frac{\pi^2 \times 210 \times 10^3 \times 306.8 \times 10^3}{\frac{\pi}{4}(50)^2 \times 350}$$

$$= \frac{6.352 \times 10^{11}}{686.875 \times 10^3}$$

$$l_e^2 = 924.76 \times 10^3$$

$$l_e = 961.9\,\text{mm}.$$

Rankine's Formula:

Empirical formula established by "Rankine".

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

$P \Rightarrow$ Crippling load by Rankine's formula.

$P_c \Rightarrow$ Crushing load $\Rightarrow \sigma_c \times A$

$\sigma_c \Rightarrow$ ultimate crushing stress

$A \Rightarrow$ c/s Area

$P_E \Rightarrow$ Crippling load by Euler's formula.

$$\Rightarrow \frac{\pi^2 EI}{l_e^2}$$

$l_e \Rightarrow$ Eff. length.

$$\frac{1}{P} \Rightarrow \frac{P_E + P_c}{P_c \cdot P_E}.$$

$$= \frac{P_c}{1 + \frac{P_c}{P_E}} \quad (\div \text{ by } P_E).$$

$P_c = \sigma_c \cdot A$

$P_E = \frac{\pi^2 EI}{l_e^2}$

$$\Rightarrow \frac{\sigma_c A}{1 + \sigma_c \cdot A / \pi^2 EI / l_e^2}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A \, l_e^2}{\pi^2 EI}}$$

$\underline{I} = A k^2$ $\quad$ $k \Rightarrow$ radius of gyration.

$$\Rightarrow \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A \, l_e^2}{\pi^2 E A k^2}}$$

$$\Rightarrow \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{\pi^2 E}\left(\frac{l_e}{k}\right)^2}$$

$$a = \frac{\sigma_c}{\pi^2 E}$$ $a \Rightarrow$ Rankine's Constant.

$$P \Rightarrow \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

S.no	Material	σ_c (N/mm²)	a
1	Wrought iron	250	1/9000
2	Cast iron	550	1/1600
3	Mildsteel	320	1/7500
4	Timber	50	1/750

P-3. The internal & external dia of hollow cast iron column are 4cm & 5cm resp. If length of this column is 3m & both end fixed determine crippling load using Rankine's formula. $\sigma_c = 550 N/mm^2$

$a = 1/1600$

G.D: $D = 5cm$ $d = 4cm$. $L = 3m$ $a = \frac{1}{1600}$

$\sigma_c = 550 N/mm^2$.

$$A = \frac{\pi}{4}\left((5)^2 - (4)^2\right)$$

$$= 2.25\pi \, cm^2 \times 100$$

$$A = 225\pi \, mm^2$$

$$I = \frac{\pi}{64}\left[(5)^4 - (4)^4\right)$$

$$= 5.7656\pi \, cm^4$$

$$= 5.765\pi \times 10^4 \, mm^4$$

$$I = 57656\pi \, mm^4$$

$$k = \sqrt{\frac{57656\pi}{225\pi}}$$

$k = 25.625mm$

$l = 3m \Rightarrow 3000mm$

$$L_e = \frac{l}{2}$$ [both ends are fixed].

$$= \frac{3000}{2}$$

$L_e = 1500mm.$

$\sigma_c = 550\ N/mm^2$ $a = \frac{1}{1600}$

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{L_e}{k}\right)^2} \Rightarrow \frac{550 \cdot 225\pi}{1 + \left(\frac{1}{1600}\right)\left(\frac{1500}{25.625}\right)}$$

$$= \frac{550 \times 225\pi}{3.1415}$$

$P = 123750N.$

$$A = \frac{\pi}{4}\left[D^2 - (0.8D)^2\right]$$

$$= \frac{\pi}{4}\left[D^2 - 0.64D^2\right]$$

$$= \frac{\pi}{4} \times 0.36D^2$$

$$A = \pi \times 0.09 D^2$$

$$I = \frac{\pi}{64}\left[(D)^4 - (0.8D)^4\right]$$

$$= \frac{\pi}{64}\left[D^4 - 0.4096 D^4\right].$$

$$= \frac{\pi}{64} \times 0.5904 D^4$$

$$I = 0.009225 \times \pi \times D^4.$$

$$I = AK^2$$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.009225 \times \pi D^4}{\pi \times 0.09 \times D^2}}$$

$$K = 0.32D$$

$$P = \frac{\sigma_c . A}{1 + a\left(\frac{L_e}{k}\right)^2}$$

$$1250000 = \frac{550 \times \pi \times 0.09 D^2}{1 + \left(\frac{1}{1600}\right) \times \left(\frac{2000}{0.32 D}\right)^2}$$

$$\frac{1250000}{550 \times \pi \times 0.09} = \frac{D^2}{1 + \frac{24414}{D^2}}$$

$$8038 = \frac{D^4}{D^2 + 24414}$$

$$8038 D^2 + 8038 \times 24414 = D^4$$

$$D^4 - 8038 D^2 - 196239700 = 0 .$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = 136.3 \text{ mm}$$

$$d = 0.8 \times 136.3 \Rightarrow 109 \text{ mm}.$$

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P6 A 1.5m long column has circular c/s of 5cm dia. One end of column is fixed in direction & position & other end is free. Calculate safe load when F.O.S is 3

a) Rankine's formula, $\sigma_c = 560 N/mm^2$ $a = \frac{1}{1600}$ for pinned end

b) Eulers formula, Young's modulus for C.I $= 1.2 \times 10^5 N/mm^2$

$l = 1.5m \Rightarrow 1500 mm$

$d = 5cm$.

$$A = \frac{\pi}{4}(5)^2 = 19.635 \, cm^2 = 19.635 \times 10^2 \, mm^2$$

$$I = \frac{\pi}{64}(5)^4 = 30.7 \, cm^4 = 30.7 \times 10^4 \, mm^4.$$

$$k = \sqrt{\frac{30.7 \times 10^4}{19.635 \times 10^2}} = 12.5 mm$$

One end is fixed & end is free.

$$L_e = 2l \Rightarrow 2 \times 1500 \Rightarrow 3000 mm.$$

a) Safe load by Rankine's formula:

$\sigma_c = 560\ N/mm^2$.

$a = 1/1600$

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{L_e}{k}\right)^2} \Rightarrow \frac{560 \times 1963.5}{1 + \left(\frac{1}{1600}\right)\left(\frac{3000}{12.5}\right)^2}$$

$P = 29708.1N$

$$\text{Safe load} = \frac{\text{Crippling load}}{F.O.S}$$

$$= \frac{29708.1}{3}$$

Safe load $= 9902.7N$

b Safe load by Euler's formula:

$$P = \frac{\pi^2 EI}{l_e^{\,2}}$$

$$= \frac{\pi^2 \times 1.2 \times 10^5 \times 30.7 \times 10^4}{(3000)^2}$$

$$P = 40200N$$

$$\text{Safe load} = \frac{40200}{3} = 1340N.$$

X

P-7 Find the Euler Crushing load for hollow cylindrical cast iron column 20 cm external dia & 25mm thick if it is 6m long & is hinged at both end. Take $E = 1.2 \times 10^5 N/mm^2$. Compare load with crushing load as given by Rankine formula. $\sigma_c = 550 N/mm^2$ & $a = \frac{1}{1600}$; for what length of column would these 2 formulae give the same crushing load?

$D = 20cm$

$t = 25mm \Rightarrow 2.5cm$.

$d = D - 2t = 20 - (2 \times 2.5) = 15cm$.

$A = \frac{\pi}{4}\left[(20)^2 - (15)^2\right] = 137.37 \ cm^2$

$A = 13737 mm^2$.

$I = \frac{\pi}{64}\left[(20)^4 - (15)^4\right] = 5366.2 \ cm^2$

$I = 53.66 \times 10^6 \ mm^4$.

$$K = \sqrt{\frac{53.66 \times 10^6}{13737}}$$

$K = 62.5 mm.$

Both end hinged $l_e = l = 6000 mm.$

$$P = \pi^2 EI / l_e^2$$

$$= \frac{\pi^2 \times 1.2 \times 10^5 \times 53.66 \times 10^6}{(6000)^2}$$

$P = 1.76 \times 10^6 N.$

Rankine's formula

$$P = \frac{\sigma_c . A}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{550 \times 13737}{1 + \left(\frac{1}{1600}\right)\left(\frac{6000}{62.5}\right)^2}$$

$$= \frac{7.5 \times 10^6}{6.76}$$

$P = 1.1 \times 10^6 N.$

$$\frac{\pi^2 EI}{l^2} = \frac{\sigma_c . A}{1 + a\left(L/k\right)^2}$$

$$\frac{\pi^2 \times 1.2\times10^5 \times 53.66\times10^6}{l^2} = \frac{550 \times 13737}{1 + \left(\frac{1}{1600}\right)\left(\frac{L}{62.5}\right)^2}$$

$$\frac{\pi^2 \times 1.2\times10^5 \times 53.66\times10^6}{550 \times 13737} = \frac{l^2}{1 + \frac{l^2}{6250000}}$$

$$8.4\times10^6 \left(1 + \frac{l^2}{6250000}\right) = l^2$$

$$8.4\times10^6 + \frac{8.4\times10^6\, l^2}{6250000} = l^2.$$

$$8.4\times10^6 + 1.34\, l^2 = l^2$$

$$1.34 l^2 - l^2 = -8.4\times10^6$$

$$0.34 l^2 = -8.4\times10^6$$

$$l = \sqrt{\frac{-8.4\times10^6}{0.34}}$$

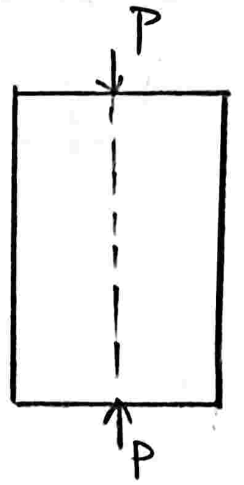
It's not possible to have same length of colum, which have same Crupling load for 2 given formulas.

Direct stress: It is Produced in a body when itbody is subjected to axial Tensile or Compressive load.

Bending stress: When body is subjected to bending moment:

When a body is subjected to axial & bending moment, then both direct & bending stress will produced in body

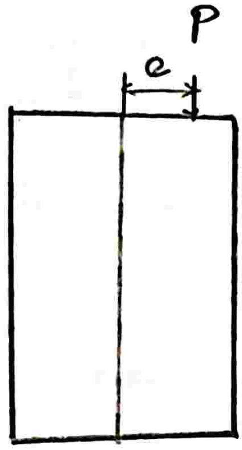
Combined bending & direct stress



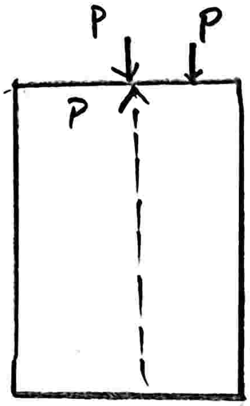
* column is subjected by compressive load P acting along axis of column. It cause direct comp. stress will uniform across c/s.

$$\sigma_0 = \frac{P}{A}$$

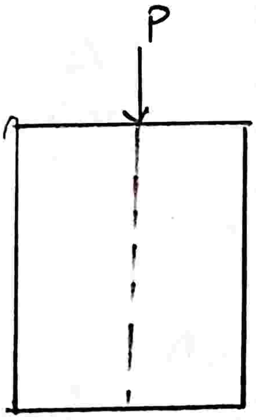
σ_0 => stress intensity
P => load
A => c/s Area



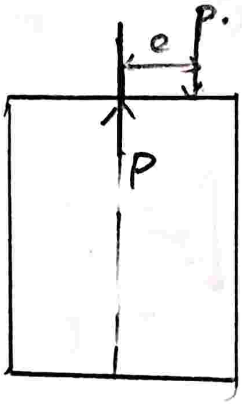
(a) Column is subjected by P whose line of action is @ distance of e from axis of column. eccentric load cause direct & bending stress.



(b) P is applied along axis of column 2 equal & opp forces P. Thus 3 force acting on column.



This force will produce direct stress.

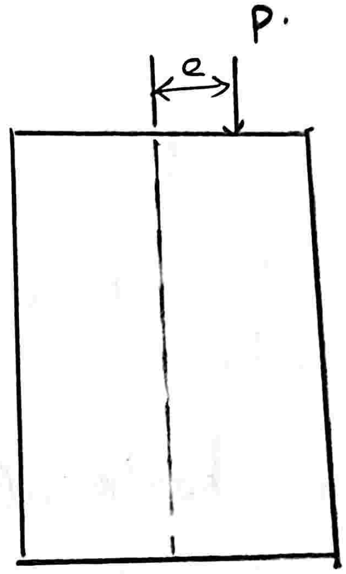


It will form couple, whose moment will $P \times e$. This couple will produce bending stress

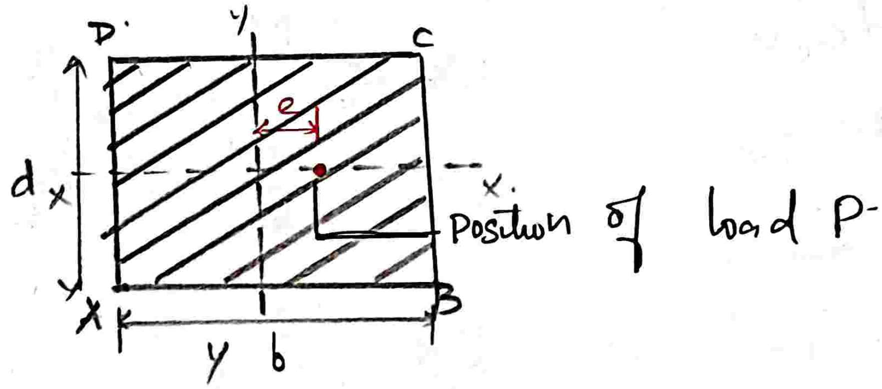
Hence eccentric load will produce direct as well as bending stress. By adding these two stresses algebrically single resultant stress can be obtained

Resultant stress when column of ▭lar section is subjected to eccentric load:

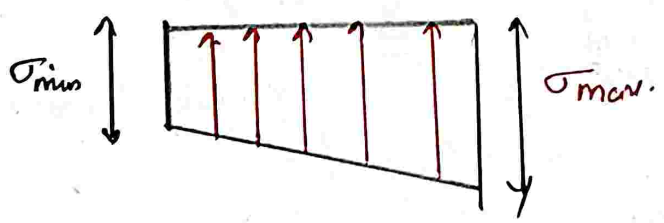
A rectangular column is subjected to eccentric -load.



a) Elevation



b) Plan



c)

Moment (M) = load x eccentricity.

$= P \times e$

direct stress (σ_0) $= \frac{P}{A}$.

$$\frac{M}{I} = \frac{\sigma_b}{\pm y}$$

$$\sigma_b = \pm y \frac{M}{I}$$

M.O.I (I) about y-y axis $= \frac{db^3}{12}$.

$$\sigma_b = \pm \frac{M}{\frac{db^3}{12}} \times y$$

$$= \pm \frac{12M}{db^3} \times y.$$

$\sigma_b \Rightarrow$ bending stress
$\sigma_0 \Rightarrow$ direct stress
$\sigma_{max} \Rightarrow$ max stress
$\sigma_{min} \Rightarrow$ min stress.

bending stress @ extreme $y = b/2$

$$\sigma_b = \pm \frac{12M}{db^3} \times \frac{b}{2}$$

($M = P \times e$).

$$= \pm 6P.e/db^2$$

$$= \pm \frac{6P.e}{d.b.b} \qquad (A = b.d)$$

$$\sigma_b = \pm \frac{6Pe}{A.b}$$

σ_{max} = direct stress + bending stress.

$$= \sigma_0 + \sigma_b$$

$$= \frac{P}{A} + \frac{6Pe}{A.b}$$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

σ_{min} = direct stress − bending stress

$$= \frac{P}{A} - \left(\frac{6Pe}{Ab}\right)$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

$$\sigma_b = \pm \frac{6Pe}{Ab} \qquad \sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right).$$

P-8 A □lar column of width 200mm & of

1 thickness 150 mm carries a pt load of 240kN

@ eccentricity of 10~~0~~mm. Determine max &

min stress.

G.P: $b = 200\,mm \quad d = 150\,mm$.

$A = 200 \times 150 = 30{,}000\ mm^2$.

$P = 240\,kN = 240 \times 10^3\ N$

$e = 10\ mm$

To find: σ_{max} & σ_{min} :

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{240 \times 10^3}{30{,}000}\left(1 + \frac{(6 \times 10)}{200}\right)$$

$$= 8 \times 1.3$$

$$\sigma_{max} = 10.4\ N/mm^2.$$

P-8 A ▭ lar column of width 200mm & of thickness 150 mm carries a pt load of 240kN @ eccentricity of 10mm. Determine max & min stress.

G.P.

$b = 200 mm \quad d = 150 mm$.

$A = 200 \times 150 = 30,000 \ mm^2$.

$P = 240 \ kN = 240 \times 10^3 \ N$

$e = 10 \ mm$

To find: σ_{max} & σ_{min} :

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{240 \times 10^3}{30,000}\left(1 + \frac{(6 \times 10)}{200}\right)$$

$$= 8 \times 1.3$$

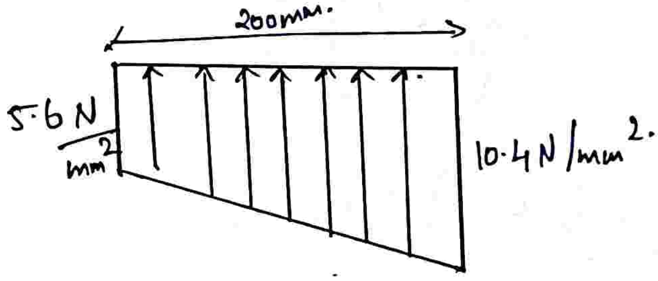
$$\sigma_{max} = 10.4 \ N/mm^2.$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{240000}{30{,}000}\left(1 - \frac{(6 \times 10)}{200}\right)$$

$$= 8 \times 0.7$$

$$\sigma_{min} = 5.6\ N/mm^2.$$



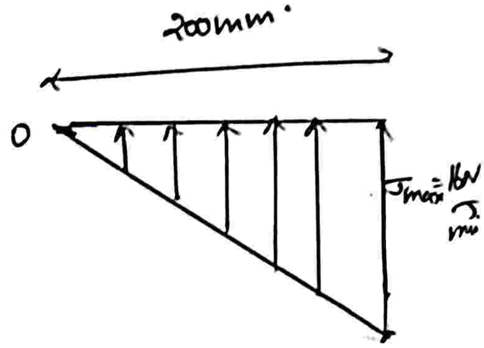
P-9 If min. stress on section is zero then find eccentricity of Point load of 240 kN acting on rectangular column. Also calculate max-stress.

$b = 200mm$, $d = 150mm$ $P = 240 \times 10^3 N$ $A = 30{,}000mm^2$.

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

$$0 = \frac{240 \times 10^3}{30{,}000}\left(1 - \frac{6e}{200}\right)$$

$$1 - \frac{6e}{200} = 0$$



$$1 = \frac{6e}{200}$$

$$e = 200/6$$

$$e = 33.33mm.$$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{240 \times 10^3}{30,000}\left(1 + \frac{(6 \times 33.33)}{200}\right)$$

$$\sigma_{max} = 16 \ N/mm^2.$$

P-10 If eccentricity is 50mm instead of 50mm, then max & min Stress..

$$\sigma_{max} = \frac{240 \times 10^3}{30,000}\left(1 + \frac{6 \times 50}{200}\right)$$

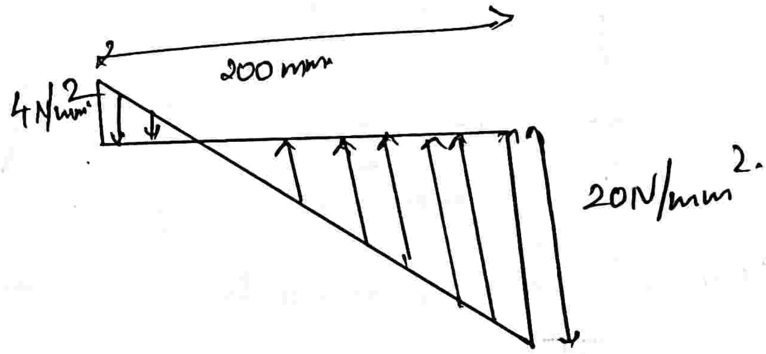
$$= 8(2.5)$$

$$\sigma_{max} = 20N/mm^2.$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{240\times10^3}{30{,}000}\left(1 - \frac{(6\times50)}{200}\right)$$

$$\sigma_{min} = -4\ N/mm^2$$



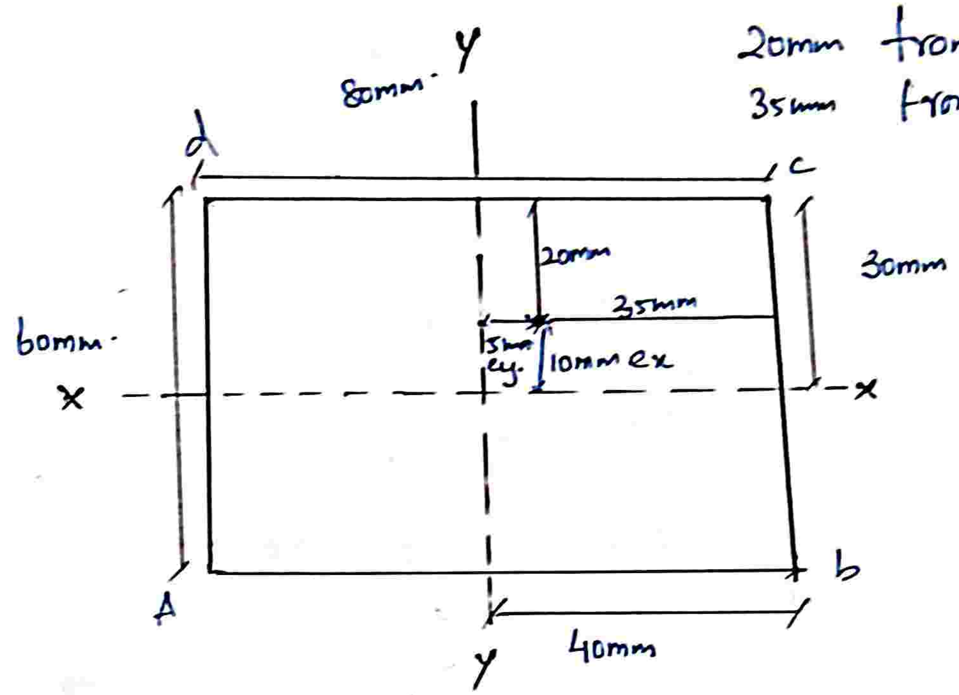
i) $e = \frac{200}{6}\ (b/6)$; $\sigma_{min} = 0$

ii) $e < b/6$; $\sigma_{min} = +ve$ When $e = 10mm < \frac{200}{6}$

iii) $e > b/6$; $\sigma_{min} = -ve$ $e = 50mm > \frac{200}{6}$.

P-11 A short column of ▭lar c/s 80mm by 60 mm carries a load of 40kN at point 20mm from longer side & 35mm from shorter side. Determin max comp & tensile stress in section

20mm from L.S
35mm from S.S.



$b = 80$ mm $d = 60$ mm $A = 80 \times 60 = 4800 mm^2$.

$e_x = 10mm$ $e_y = 5mm$.

$M_x = P \times e_x = 40 \times 10^3 \times 10 = 400 \times 10^3$ N.mm.

$M_y = P \times e_y = 40 \times 10^3 \times 5 = 200 \times 10^3$ N.mm.

$$I_{xx} = \frac{bd^3}{12} = \frac{80(60)^3}{12} = 1.44 \times 10^6 \ mm^4.$$

$$I_{yy} = \frac{db^3}{12} = \frac{60(80)^3}{12} = 2.56 \times 10^6 \ mm^4.$$

i) max. Comp. stress at C x & y are +ve.

$x = 40mm$ $y = 30mm$

$$= \frac{P}{A} + \frac{M_y\, x}{I_{yy}} + \frac{M_x \cdot y}{I_{xx}}$$

$$= \frac{40 \times 10^3}{4800} + \frac{400 \times 10^3 \times 40}{2.56 \times 10^6} + \frac{400 \times 10^3 \times 30}{1.44 \times 10^6}$$

$$= 8.33 + 3.125 + 8.33$$

max σ_n Comp. Stress $= 19.785\ N/mm^2$.

ii max Tensile stress at A $x = -40mm\ y = -30\ mm$.

$$= \frac{40 \times 10^3}{4800} - \frac{200 \times 10^3 \times 40}{2.56 \times 10^6} - \frac{400 \times 10^3 \times 30}{1.44 \times 10^6}$$

$$= 8.33 - 3.125 - 8.33$$

$$= -3.125\ N/mm^2$$

i) If $\sigma_0 = \sigma_b$, Tensile stress will be zero

ii) $\sigma_0 > \sigma_b$, stress will be compressive

iii) $\sigma_0 < \sigma_b$, stress will be Tensile

iv) $\sigma_0 \geq \sigma_b$, no tensile stress.

e must be less than $b/6$. If load is applied any distance less than $b/6$ from axis, stress are wholly compressive.

Hence the range within which load can be applied so as not to produce any Tensile stress is within in the middle third of base.

i) If $\sigma_0 = \sigma_b$, Tensile stress will be zero

ii) $\sigma_0 > \sigma_b$, stress will be compressive

iii) $\sigma_0 < \sigma_b$, stress will be Tensile

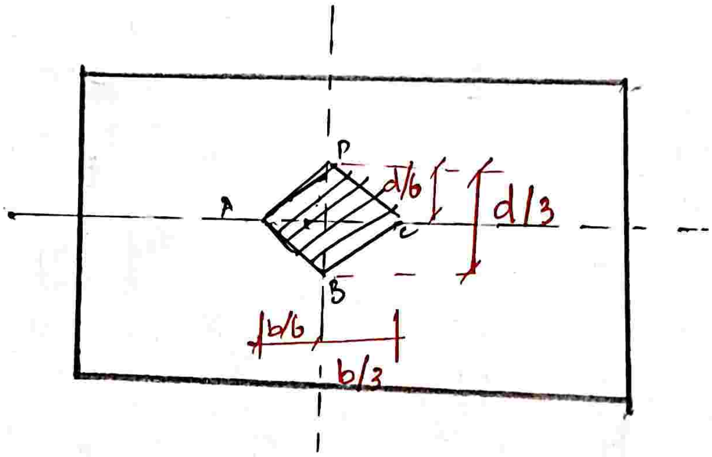
iv) $\sigma_0 \geq \sigma_b$, no tensile stress.

e must be less than b/6. If load is applied any distance less than b/6 from axis, stress are wholly compressive.

Hence the range within which load can be applied so as not to produce any Tensile stress is within in the middle third of base.

Middle Third rule for ▭ ar section i.e Kernel of section.

The load is applied at any part of section so that it doesn't produce Tensile stress called core or kernel of section.



$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

If σ_{min} is $-ve$, then stress will be Tensile.

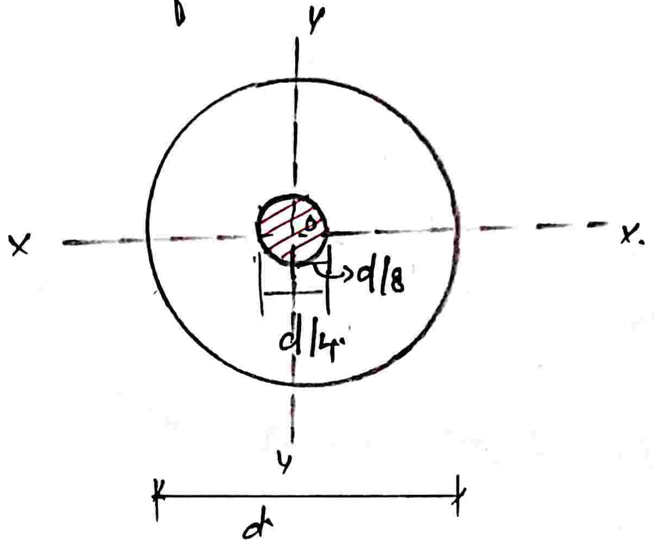
$\sigma_{min} \geq 0$

$$\frac{P}{A}\left(1 - \frac{6e}{b}\right) \geq 0$$

$1 \geq \frac{6e}{b}$ $\qquad \frac{b}{6} \geq e$

$e \geq b \times 6$ $\qquad e \leq b/6$.

Middle Quarter Rule for Circular Section
[ie kernel of Section]:



$$A = \frac{\pi}{4} d^2$$

$$\sigma_0 = \frac{P}{A} = \frac{4P}{\pi d^2}$$

$$M = P \times e$$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{P \times e \times y}{I} \qquad y = d/2$$

$$\sigma_b = \frac{M}{I} (\pm d/2)$$

$$= \pm \frac{P \times e \times d/2}{\pi/64 \, d^4} = \pm \frac{32 \, Pe}{\pi d^3}$$

$$\sigma_b = \pm \frac{32 P e}{\pi d^3}$$

$$\sigma_{min} = \sigma_o - \sigma_b$$

$$= \frac{4P}{\pi d^2} - \frac{32 P e}{\pi d^3}$$

For No Tensile stress $\sigma_{min} \geq 0$

$$\frac{4P}{\pi d^2} - \frac{32 P e}{\pi d^3} \geq 0$$

$$\frac{4P}{\pi d^2}\left(1 - \frac{8e}{d}\right) > 0$$

$$1 - \frac{8e}{d} \geq 0$$

$$1 \geq \frac{8e}{d}$$

$$\frac{d}{8} \geq e$$

$$\boxed{e \leq d/8}$$

e must be less than or equal to d/8

It means load can be eccentric on any side of centre of circle by amount equal to d/8. Thus l.o.a of load within circle of dia equal to 1/4 of main circle, than stress will compressive throughout the section

* Kernel of Hollow circular section

$$\text{Kernel dia} = \frac{D_0^2 + D_i^2}{4D_0}$$

* Kernel of Hollow □lar section:

$$\text{Along } x \text{ axis} = \frac{BD^3 - bd^3}{6D(BD - bd)}$$

$$\text{Along } y \text{ axis} = \frac{DB^3 - db^3}{6B(BD - bd)}$$

Unit - II

Deflection of Determinate Beam

Governing differential equation - Elastic curve for various types of beams - Slope and deflection Macaulay's method - Moment Area Method - conjugate beam method.

Beam : It is horizontal structural member which resist vertical force, shear force & B.M.

Statically Indeterminate structure : The structure can't be solved using equilibrium equation alone.

$$U > E_E.$$

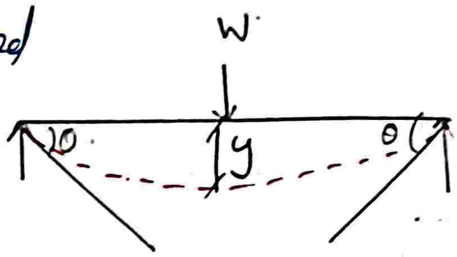
$U \Rightarrow$ no. of unknown force

$E_E \Rightarrow$ equilibrium equation

Statically Determinate structure : The structure that can be solved using equilibrium eqn alone

$$U = E_E$$

Deflection: It is vertical distance of beam measured before & after loading



It is denoted by "y".

Deflection at support is always zero

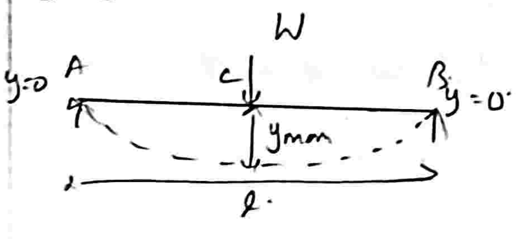
Slope: It is angle in radian measured b/w tangent to elastic curve & original axis of beam.

Slope is denoted by "θ" or $\frac{dy}{dx}$.

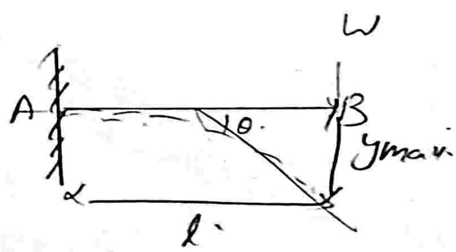
It's unit is radian.

Boundary condition:

i) S.S.B

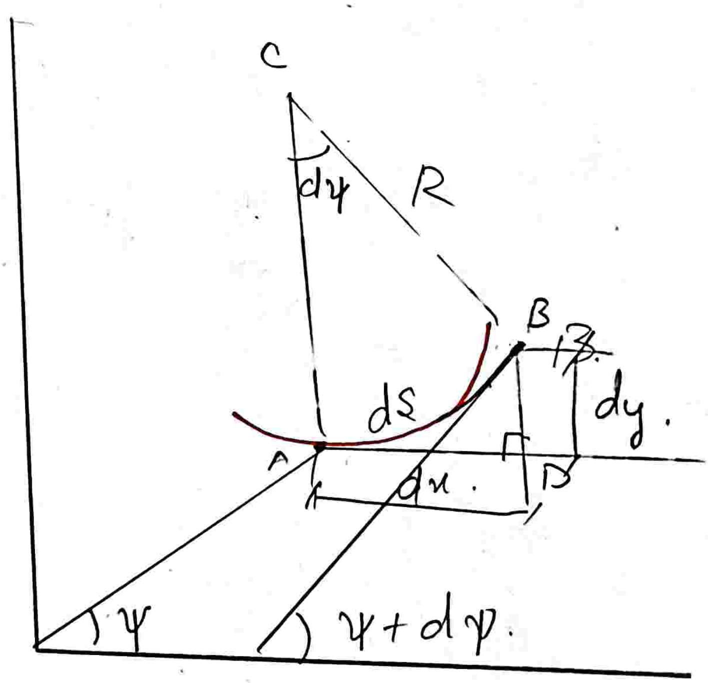


ii Cantilever beam



i) @ A & B ; $y = 0$	i) @ A ; $y = 0$
ii) @ C ; $y = max$ -	ii) @ A ; $\theta = 0$
iii) @ C ; $\theta = 0$	iii) @ A & B $y = max$
iv) @ A & B ; $\theta = max$ -	iv @ B ; slope also max -

Expression for differential equation for deflection - slope & deflection of beam:



Let ds = Small length of bent beam considered

C = Centre for bent beam arc.

R = Radius of curvature of beam AB

ψ = Angle made by pt A of beam with horizontal

$\psi + d\psi$ = Angle made by pt B of beam with horizontal

$d\psi$ = Angle made by beam AB w.r.t centre C.

$\angle ACB = d\psi$

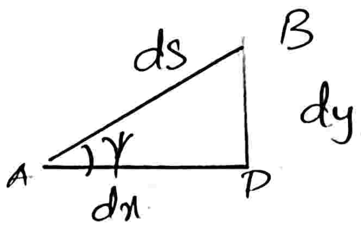
length of arc AB $= ds = d\psi \times R$.

$$\frac{ds}{R} = d\psi$$

$$\frac{1}{R} = \frac{d\psi}{ds} \quad \longrightarrow (1)$$

5

Now considering Right angle ΔADB.



$$\tan\psi = \frac{BD}{AD} = \frac{dy}{dx}.$$

$$\tan\psi = \frac{dy}{dx} \rightarrow (2).$$

$$\cos\psi = \frac{AD\ (\text{adj})}{AB\ (\text{hyp})} = \frac{dx}{ds}$$

$$\cos\psi = \frac{dx}{ds} \Rightarrow (3)$$

$\sec\theta = \frac{1}{\cos\theta}$.

$$\sec\psi = \frac{ds}{dx} \Rightarrow (4)$$

differentiate eq (2) w.r.t x,

$$\sec^2\psi \cdot \frac{d\psi}{dx} = \frac{d^2y}{dx^2}$$

$$\sec^2\psi \frac{d\psi}{ds} \cdot \frac{ds}{dx} = \frac{d^2y}{dx^2}.$$

$$\sec^2\psi \cdot \frac{d\psi}{ds} \sec\psi = \frac{d^2y}{dx^2}$$ => Refer eq 4

$$\sec^3\psi \cdot \frac{1}{R} = \frac{d^2y}{dx^2}$$ => Refer eq 1.

$$\frac{1}{R} = \frac{d^2y/dx^2}{\sec^3\psi}$$

$$\frac{1}{R} = \frac{d^2y/dx^2}{(\sec^2\psi)^{3/2}} \Rightarrow \frac{d^2y/dx^2}{(\tan^2\psi+1)^{3/2}}.$$

$$\sec^3\psi = \sec^{\cancel{2}}\psi^{3/\cancel{2}} \Rightarrow \sec^3\psi.$$

$$\sec^2\theta = \tan^2\theta + 1.$$

$$\frac{1}{R} = \frac{d^2y/dx^2}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$ => refer 2.

$\therefore$ denominator is very very smal

neglecting term $(dy/dx)^2$ we get.

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} \rightarrow (6)$$

from 5 & 6

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$M = EI \cdot \frac{d^2y}{dx^2} \rightarrow (7)$$

from eq. 7 we get differential eqn from which we can calculate Slope & deflection by using Macaulay's method, Double integration method

$$M = EI \frac{d^2y}{dx^2}$$

differentiate M w.r.t x.

$$\frac{dM}{dx} = EI \frac{d^3y}{dx^3}$$

$$F = EI \frac{d^3y}{dx^3}$$ => Shearforce

differentiate F w.r.t x

$$\frac{dF}{dx} = EI \frac{d^4y}{dx^4}$$

$$\omega = EI \frac{d^4y}{dx^4}$$ => rate of loading.

Deflection $= y$

Slope $= dy/dx$

Moment $= EI\, d^2y/dx^2$

shear Force $= EI\, d^3y/dx^3$

Rate of loading $= EI\, d^4y/dx^4$.

$E \Rightarrow N/mm^2$

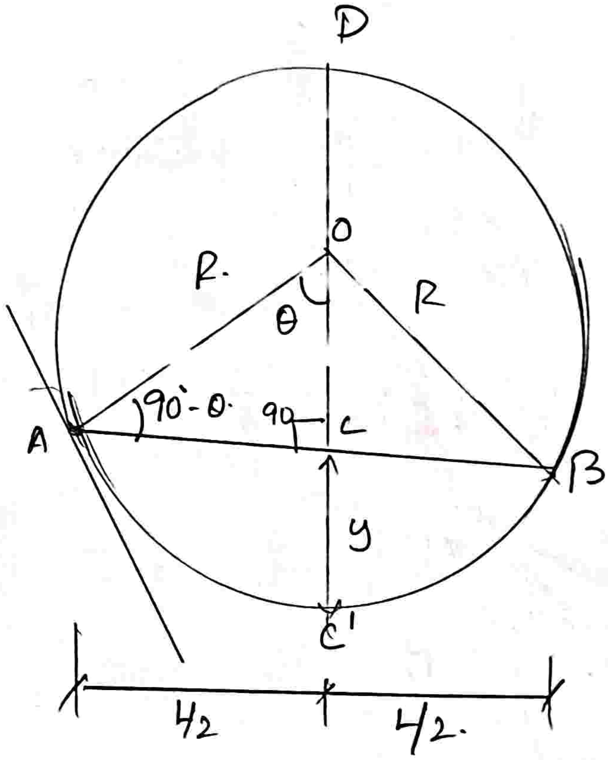
$I \Rightarrow mm^4$

$M \Rightarrow Nm$.

$y \Rightarrow mm$

$x \Rightarrow m$.

Deflection in slope of beam subjected to uniform bending:



A beam of length is subjected to uniform bending moment. , hence it winds in to circular arc ACB → AC'B.

R ⇒ Radius of curvature

y ⇒ deflection of beam at centre

I ⇒ M.O.I

E ⇒ Young's modulus

θ ⇒ slope of beam @ ends

$\tan\theta = \theta$

$$\frac{dy}{dx} = \tan\theta = \theta .$$

from geometry of circle

$AC \times CB = DC \times CC'$

$$\frac{L}{2} \times \frac{L}{2} = (2R - y) \times y$$

$DC = DC' - CC'$

$DC = 2R - y$

$$\frac{L^2}{4} = 2Ry - y^2 .$$

y is small, neglecting y^2

$$\frac{L^2}{4} = 2Ry$$

$$y = \frac{L^2}{8R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$R = \frac{E \times I}{M}$$

$$y = \frac{L^2}{8 \times \frac{EI}{M}}$$

$$y = \frac{ML^2}{8EI}$$

From ΔAOB

$$\sin\theta = \frac{AC}{AO} = \frac{L/2}{R} = \frac{L}{2R}.$$

θ is very small $\sin\theta = \theta$.

$$\theta = L/2R.$$

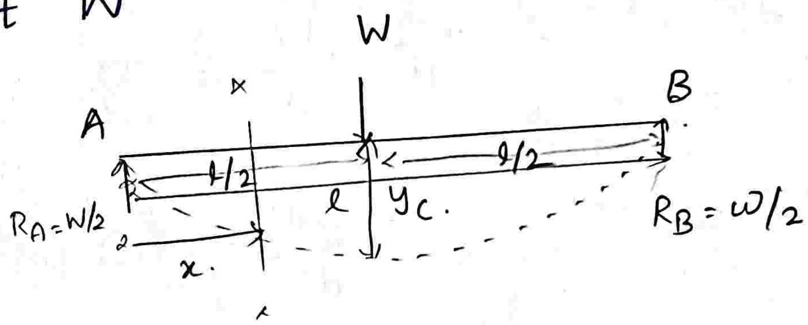
$$= \frac{L}{2 \times \frac{EI}{M}}$$

$$\theta = \frac{ML}{2EI}$$

θ @ A and B

Deflection of Simply Supported Beam carrying a Point load at centre.

A SSB AB of length L and carrying a pt W @ centre.



$$R_A = R_B = W/2$$

$$M_x = R_A \cdot x$$

$$M_x = \frac{W}{2} \cdot x$$

$$M = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = \frac{W}{2} \cdot x \quad \rightarrow (1)$$

On integrating eqn (1).

$$EI \cdot \frac{dy}{dx} = \frac{W}{2} \frac{x^2}{2} + c_1 \quad \rightarrow (2)$$

max deflection $x = l/2$, $\frac{dy}{dx} = 0$

$$EI \cdot 0 = \frac{W}{4}\left(l/2\right)^2 + C_1$$

$$0 = \frac{W}{4}\frac{l^2}{4} + C_1$$

$$C_1 = -\frac{Wl^2}{16}$$

Sub C_1 in eqn (2)

$$EI \cdot \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16} \rightarrow (3)$$

Slope max @ A $x = 0$

$$EI\left(\frac{dy}{dx}\right)_A = \frac{W}{4}(0) - \frac{Wl^2}{16}$$

$$EI\,\theta_A = \frac{-WL^2}{16}$$

$$\theta_B = \theta_A = \frac{-WL^2}{16EI}$$

Integrating eqn (3).

$$EI \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16}$$

$$EI.y = \frac{W}{4}\frac{x^3}{3} - \frac{WL^2}{16}x + C_2 \quad \to (4)$$

$x = 0 \quad y = 0.$

$$EI \times 0 = 0 - 0 + C_2$$

$$C_2 = 0$$

$$EI.y = \frac{Wx^3}{12} - \frac{WL^2 x}{16}$$

$y = y_c$ @ $x = L/2$

$$EI \times y_c = \frac{W}{12}\left(\frac{L}{2}\right)^3 - \frac{WL^2}{16}\left(\frac{L}{2}\right)$$

$$= \frac{WL^3}{96} - \frac{WL^3}{32} \Rightarrow \frac{WL^3 - 3WL^3}{96}$$

$$= \frac{-2WL^3}{96} = -\frac{WL^3}{48}$$

$$y_c = -WL^3/48EI$$

downward deflection $y_c = WL^3/48EI$

P-1 A beam 4m long S.S at its ends, Carries a Pt load W @ its centre. If Slope at end of beam is not to exceed 1°. find deflection at centre of beam.

$L = 4m \Rightarrow 4000\,mm.$

$\theta_A = \theta_B = 1° = \frac{1 \times \pi}{180} = 0.01745$ radian

$$\theta_A = \frac{WL^2}{16EI}$$

$$0.01745 = \frac{WL^2}{16EI}$$

$$y = \frac{WL^3}{48EI}$$

$$= \frac{WL^2}{16EI} \times \left(\frac{L}{3}\right)$$

$$= 0.01745 \times \left(\frac{4000}{3}\right)$$

$y = 23.26\,mm.$

P-2 A beam of 6 m long S.S @ its end is carrying a pt load of 50kN @ it centre. The M.O.I is 78×10^6 mm^4. If E is 2.1×10^5 N/mm^2. calculate
i) deflection at centre of beam
ii) slope at support.

i) $y = \frac{WL^3}{48EI} \Rightarrow \frac{50 \times 10^3 \times (6000)^3}{48 \times 2.1 \times 10^5 \times 78 \times 10^6}$

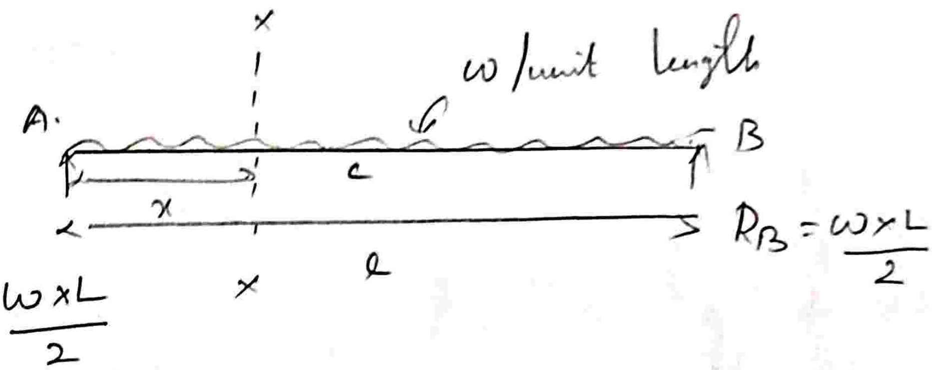
$y = 13.73$ mm.

ii) $\theta = WL^2/16EI = \frac{50 \times 10^3 (6000)^2}{16 \times 2.1 \times 10^5 \times 78 \times 10^6}$

$= 6.8 \times 10^{-3}$ rad

$\theta = 3.84°$

Deflection of S.S.B with UDL.



$$R_A = R_B = \frac{wl}{2}$$

$$M_x = R_A \cdot x - w \cdot x \times \frac{x}{2}$$

$$M_x = \frac{wl}{2} \cdot x - \frac{wx^2}{2}$$

$$EI \cdot \frac{d^2y}{dx} = \frac{wl}{2} \cdot x - \frac{wx^2}{2}$$

Integrating above eqn

$$EI \cdot \frac{dy}{dx} = \frac{wl}{2} \frac{x^2}{2} - \frac{w}{2} \cdot \frac{x^3}{3} + C_1. \quad \to (1)$$

Integrating above eqn (1)

$$EI \cdot y = \frac{wl}{4} \frac{x^3}{3} - \frac{w}{6} \frac{x^4}{4} + C_1 x + C_2 \quad \to (2)$$

bc

i) $x=0, \; y=0$ ii) $x=L, \; y=0$

Sub b.c (i) in eqn ②.

$$EIy = \frac{wL}{4}\frac{x^3}{3} - \frac{w}{6}\frac{x^4}{4} + c_1x + c_2$$

$$0 = 0 - \frac{w}{6}\frac{x^4}{4} + c_2$$

$$\boxed{c_2 = 0}$$

Sub bc ② in eqn ②.

$$0 = \frac{wL(L)^3}{12} - \frac{w}{\cancel{24}\,24}(L)^4 + c_1L + 0$$

$$0 = \frac{wL^4}{12} - \frac{wL^4}{24} + c_1L$$

$$c_1L = \frac{-wL^4}{12} + \frac{wL^4}{24}$$

$$c_1 = \frac{-wL^3}{12} + \frac{wL^3}{24}$$

$$c_1 = \frac{-wl^3}{24}$$

Sub c in eqn ① in ②

$$EI \cdot \frac{dy}{dx} = \frac{w.L}{4} x^2 - \frac{w}{6} x^3 - \frac{wL^3}{24}$$

$\hookrightarrow$ ③

$$EI \cdot y = \frac{wLx^3}{12} - \frac{w}{24} x^4 + \left(\frac{-wL^3}{24}\right) + 0$$

$$EI \cdot y = \frac{w.L}{12} x^3 - \frac{w}{24} x^4 - \frac{wL^3}{24} x.$$

$\hookrightarrow$ ④.

@ $x = 0$ $\frac{dy}{dx} = \theta_A$. Sub value in ③.

$$EI. \theta_A = \frac{wL}{4} \times 0 - \frac{w}{6} \times 0 - \frac{wL^3}{24}$$

$$= \frac{-wL^3}{24} \Rightarrow \frac{-WL^2}{24}$$

$$\theta_A = \frac{-WL^2}{24 EI}$$

$$\theta_B = \frac{-WL^2}{24 EI}$$

@ pt C $x = l/2$ $y = y_c$ in eqn (4)

$$EI \cdot y_c = \frac{w \cdot L}{12}\left(\frac{L}{2}\right)^3 - \frac{w}{24}\left(\frac{L}{2}\right)^4 - \frac{wL^3}{24}\frac{L}{2}$$

$$= \frac{wL^4}{96} - \frac{wL^{\cancel{3}4}}{384} - \frac{wL^4}{48}$$

$$= \frac{-5wL^4}{384}$$

$$y_c = \frac{-5wL^{\cancel{3}4}}{384EI}$$

$$y_c = \frac{-5WL^3}{384EI}.$$

Downward deflection.

$$y_c = \frac{5WL^3}{384EI}$$

P3 A beam of length 5m cy uniform □lar section is Supported at its end cy Carry UDL over entire length calculate depth of section if max Permissible bending Stress is $8 N/mm^2$ cy central deflection is not to exceed 10mm.

$E = 1.2 \times 10^4 \ N/mm^2$

$L = 5m = 5000mm.$

$f = 8 N/mm^2 \qquad y_c = 10mm.$

$W =$ Total load $\qquad d =$ depth of beam.

$$M = \frac{wL^2}{8} = \frac{WL}{8} \quad (W = w.L)$$

$$\frac{M}{I} = \frac{f}{y}.$$

$$M = \frac{f}{y} I \Rightarrow \frac{8 \times I}{d/2}$$

$$M = \frac{16 I}{d}$$

$$\frac{W.L}{8} = \frac{16T}{d}$$

$$W = \frac{16 \times 8 I}{L d}$$

$$W = \frac{128 I}{L d}$$

$$y_c = \frac{5}{384} \frac{W L^3}{E I}$$

$$10 = \frac{5}{384} \times \frac{128 I}{L d} \times \frac{L^3}{E I}$$

$$10 = \frac{5}{384} \times \frac{128 \times \cancel{I} \times L^2}{d \times E}$$

$$d = \frac{5 \times 128 \times (5000)^2}{384 \times 10 \times 1.2 \times 10^4}$$

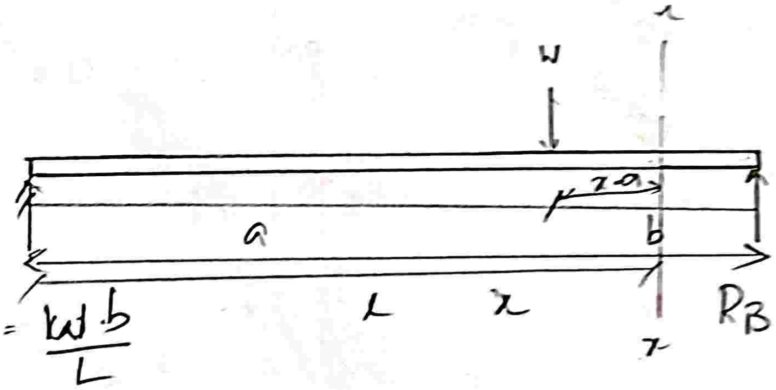
$$d = 347.2 mm.$$

Macaulay's Method:

* This method was devised by Mr. M.H. Macaulay and is known as Macaulay's method.

* It consists in special manner in which B.M at any section is expressed as in manner in which integrations are carried out.

Deflection of S.S.B with eccentric point load



$R_A = \frac{W \cdot b}{L}$ $\quad R_B = \frac{W \cdot a}{L}$

$$M_x = R_A \cdot x - W(x-a)$$

$$= \frac{Wb}{L} \cdot x \;|\; - W(x-a)$$

$$M = EI \cdot \frac{d^2y}{dx^2}$$

$$EI \cdot \frac{d^2y}{dx^2} = \frac{W.b}{L}x \;\Big|\; -W(x-a) \rightarrow ①$$

$$EI \frac{dy}{dx} = \frac{Wb}{L}\frac{x^2}{2} + c_1 \;\Big|\; - \frac{W(x-a)^2}{2}$$

↳ ②

$$EI.y = \frac{W.b}{2L}\frac{x^3}{3} + c_1.x + c_2 \;\Big|\; - \frac{W}{2}\frac{(x-a)^3}{3}$$

↳ ③

@A $x=0,\ y=0$ in eqn ③ upto dotted line

$$0 = 0 + 0 + c_2$$

$$\boxed{c_2 = 0}$$

@B $x=L,\ y=0$ in eq ③

$$0 = \frac{Wb}{2L}\frac{L^3}{3} + c_1 L + 0 - \frac{W}{2}\frac{(L-a)^3}{3}$$

$(L-a=b)$

$$c_1 L = \frac{W}{6}b^3 - \frac{WbL^2}{6}$$

$$c_1 L = \frac{-W.b}{6}(L^2-b^2)$$

$$C_1 = \frac{-W \cdot b}{6L}(L^2 - b^2)$$

Sub c_1 in eq (2)

$$EI \cdot \frac{dy}{dx} = \frac{Wb}{L} \cdot \frac{x^2}{2} + \left[-\frac{Wb}{6L}(L^2 - b^2\right] \Big| - \frac{W(x-a)^2}{2}$$

$\frac{dy}{dx} = \theta_A$, $x = 0$. upto dotted line.

$$EI \cdot \theta_A = 0 - \frac{Wb}{6L}(L^2 - b^2)$$

$$\theta_A = \frac{-Wb}{6EIL}(L^2 - b^2) \rightarrow (4)$$

Sub C_1 & C_2 in eq (3).

$$EI \cdot y = \frac{Wb}{6L}x^3 + \left[-\frac{Wb}{6L}(L^2 - b^2)\right]x + 0 \Big| - \frac{W}{6}(x-a)^3$$

$y = y_c$, $x = a$. upto dotted line

$$EI \cdot y_c = \frac{Wb}{6L}a^3 - \frac{Wb}{6L}[L^2 - b^2]a$$

$$= \frac{Wb}{6L} \cdot a \left[a^2 - L^2 + b^2\right]$$

$$= -\frac{Wab}{6L} \left(L^2 - a^2 - b^2\right)$$

$$= \frac{-Wab}{6L} \left[(a+b)^2 - a^2 - b^2\right]$$

$$= \frac{-Wab}{6L} \left[a^2 + b^2 + 2ab - a^2 - b^2\right]$$

$$= -\frac{Wab}{6L} \left[2ab\right].$$

$$y_c = -\frac{Wa^2b^2}{3EIL}$$

$$y = \frac{A\bar{x}}{EI}$$

$$\bar{x} = \frac{2}{3} \cdot \frac{l}{2} \Rightarrow l/3$$

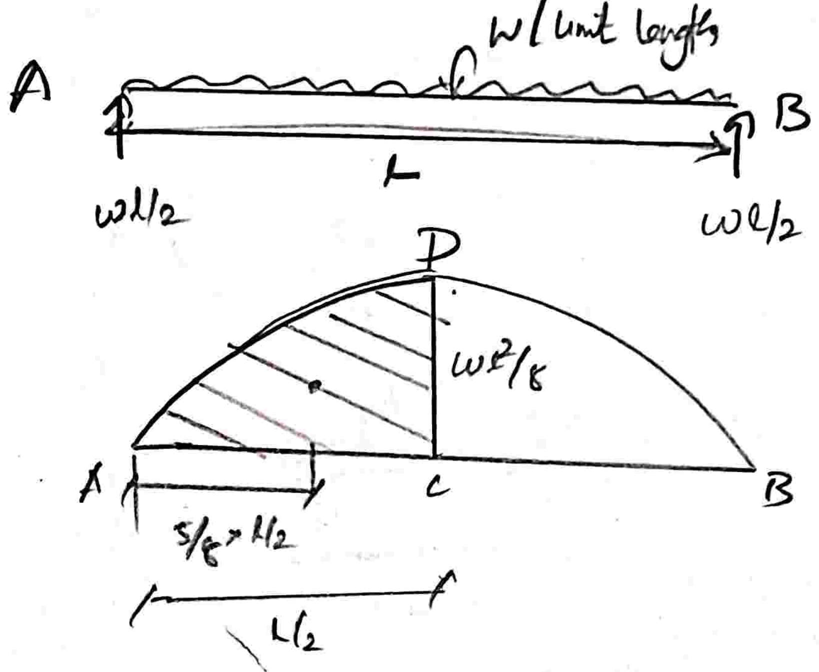
$$y = \frac{\frac{Wl^2}{16} \cdot \frac{l}{3}}{EI}$$

$$y = \frac{Wl^3}{48EI}$$

$$\theta_A = \frac{Wl^2}{16EI}$$

$$y = \frac{Wl^3}{48EI}$$

Slope And Deflection Of S.S.B Carrying UDL by Mohr's Theorem



Slope at A = Area of BMD AC /EI

Area of BMD A to C $= \frac{2}{3} \times AC \times CD$

$$= \frac{2}{3} \times \frac{l}{2} \times \frac{wl^2}{8}$$

$$= wl^3/24.$$

$$\theta_A = \frac{wl^2}{24EI}$$

$$y = \frac{A\bar{x}}{EI}$$

$$\bar{x} = \frac{5}{8} \times Ac = \frac{5}{8} \times \frac{l}{2}$$

$$\bar{x} = \frac{5l}{16}$$

$$= \frac{\frac{wl^3}{24} \times \frac{5l}{16}}{EI}$$

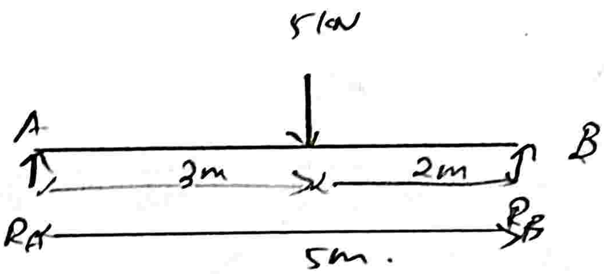
$$y = \frac{5 w.l^4}{384 EI}$$

$$\theta = \frac{wL^3}{24EI}$$

$$y = \frac{5wl^4}{384 EI}$$

Conjugate Beam method:

A SSB of length 5m carries Point load of 5kN at distance of 3m from left end If $E = 2\times10^5\ N/mm^2$ and $I = 10^8\ mm^4$. Determine Slope at left support and deflection under Point using conjugate beam method.



$E = 2\times10^5\ N/mm^2$ $\qquad\qquad$ $I = 10^8\ mm^4$

$= 2\times10^5\ \dfrac{N}{(10^{-3})^2 m^2}$ $\qquad\qquad$ $= 10^8 \times (10^{-3})^4 m^4$

$= 2\times10^{11}\ N/m^2$ $\qquad\qquad$ $I = 10^{-4} m^4.$

$R_A + R_B = 5$

$\Sigma M_A = 0.$

$-R_B\times5 + (5\times3) = 0$

$-5R_B = -15$

$R_B = 3kN$.

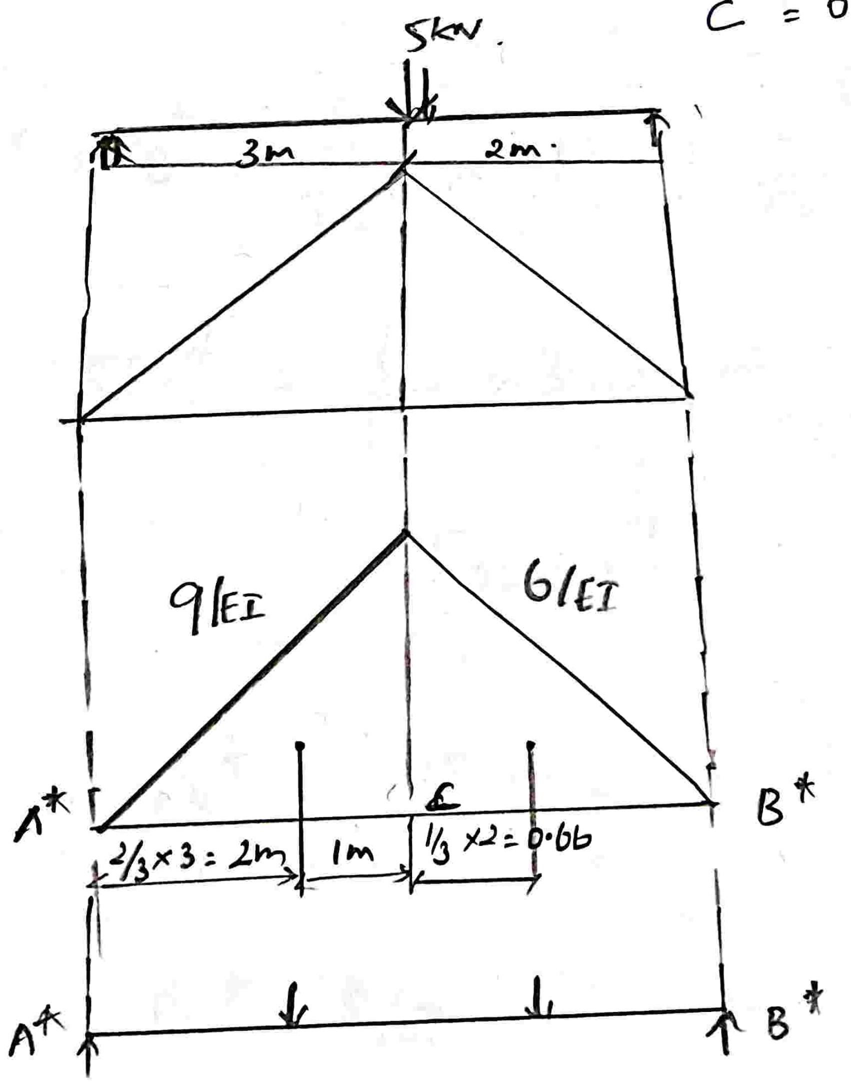
$R_A = 5-3 = 2kN$.

$R_A = 2kN$.

B.M @ A = 0 , B = 0, $C = R_B \times 2$

$= 3 \times 2$

$C = 6 \ kN$.



$$R_A^* + R_B^* = \left[\frac{1}{2} \times 3 \times \frac{6}{EI}\right] + \left[\frac{1}{2} \times 2 \times \frac{6}{EI}\right]$$

$$= \frac{9}{EI} + \frac{6}{EI}$$

$$R_A^* + R_B^* = 15/EI$$

Taking moment about A^* $\quad \Sigma M_A^* = 0.$

$$-5 \times R_B^* + \frac{6}{EI} \times 3.66 + \frac{9}{EI} \times 2 = 0.$$

$$5R_B^* = \frac{6}{EI} \times 3.66 + \frac{9}{EI} \times 2$$

$$= \frac{1}{EI}\left[22 + 18\right]$$

$$R_B^* = \frac{40}{5EI}$$

$$R_B^* = 8/EI$$

$$R_A^* = \frac{15}{EI} - \frac{8}{EI}$$

$$R_A^* = 7/EI$$

Slope at A (θ_A) $\theta_A = 7/EI$

$$= 7 / 2\times10^8 \times 10^{-4}$$

$$= 0.0035 \text{ radian.}$$

Slope at C (θ_C) : $\frac{7}{EI} - \frac{9}{EI} = \frac{-2}{EI}$

$= 2/EI$ (counter clockwise)

$= 2/2\times10^8 \times10^{-4} = 0.0001$ rad

Slope @ B (θ_B) $= \frac{-2}{EI} - 6/EI$

$$= -8/EI$$

$$= 8 / 2\times10^8 \times10^{-4}$$

$$= 0.0004 \text{ rad}$$

B.M @ C

$$= \left(\frac{8}{EI} \times 2\right) - \left(\frac{6}{EI} \times 0.66\right)$$

$$= \frac{16}{EI} - \frac{4}{EI}$$

$$= \frac{1}{EI}(16-4)$$

$$= 12/EI$$

$$= 12/2 \times 10^{-8} \times 10^{-4}$$

$B.M_c = 0.6mm.$

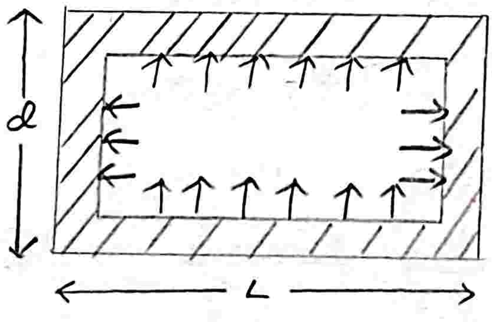
UNIT - III

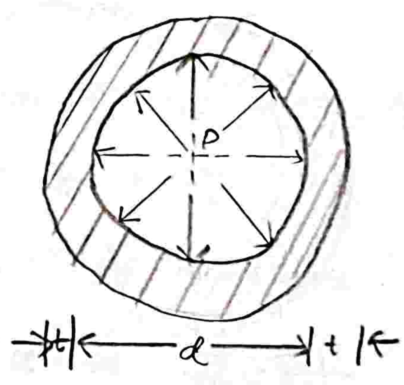
CYLINDERS

Introduction :

The vessels such as boilers, compressed air receivers etc, are of cylindrical and spherical forms. These vessels are generally used for storing fluids under pressure. The walls of such vessels are thin as compared to their diameters. If the thickness of the wall of the cylindrical vessel is less than $\frac{1}{15}$ to $\frac{1}{20}$. In case of thin cylinders, the stress is assumed over the thickness of the wall.

Thin cylindrical vessel subjected to Internal Pressure :





Let,

d = Internal diameter of thin cylinder

t = Thickness of wall of the cylinder

P = Internal pressure of fluid

L = Length of the cylinder.

Stresses in a Thin cylindrical vessel subjected to Internal Pressure:

When a thin cylindrical vessel is subjected to internal fluid pressure, the stress in the wall of the cylinder on the cross-section along the axis and the cross-section to the axis are set up.

1) Circumferential stress and
2) Longitudinal stress.

Expression for circumferential stress (or Hoop stress)

Consider a thin cylindrical vessel subjected to an internal fluid pressure. The circumferential stress will be set up in the material of cylinder.

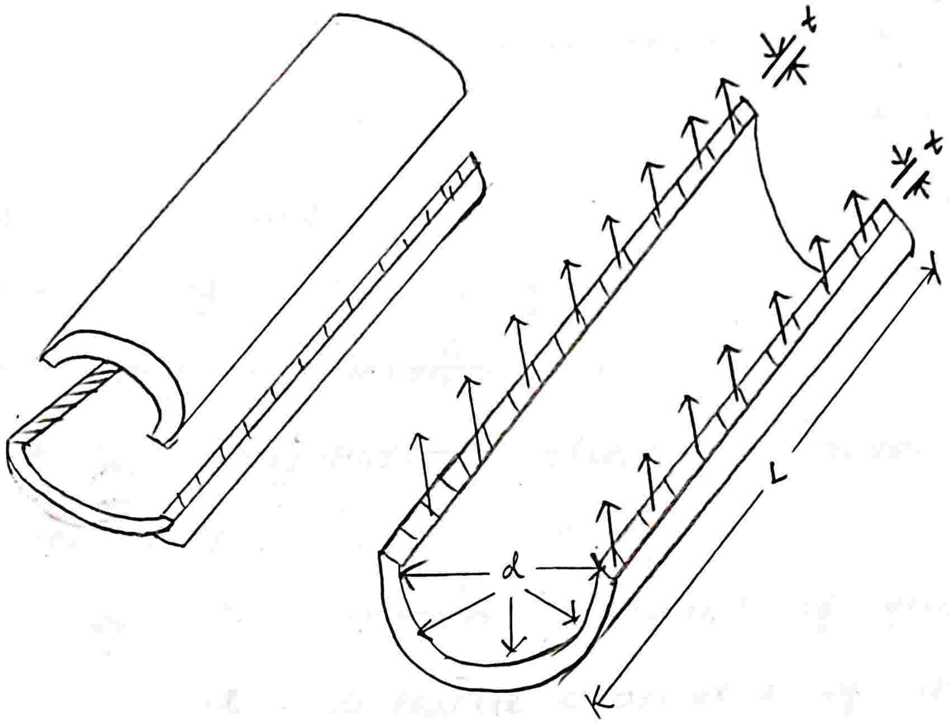
The expression for hoop stress or circumferential stress (σ_1) is obtained as given below.

Let, P - Internal pressure of fluid

d - Internal diameter of cylinder

t - Thickness of wall of cylinder

σ_1 - circumferential or hoop stress in the material.



Force due to fluid pressure } = p × Area on which p is acting

$= p \times (d \times L)$

Force due to circumferential stress

$= \sigma_1 \times$ Area on which σ_1 is acting

$= \sigma_1 \times (L \times t + L \times t)$

$= \sigma_1 \times 2Lt = 2\sigma_1 \times L \times t$

Equating ① & ②, we get

$$p \times d \times L = 2\sigma_1 \times L \times t$$

$$\sigma_1 = \frac{pd}{2t}$$

The stress is tensile.

Expression for longitudinal stress:

consider a thin cylindrical vessel subjected to internal fluid pressure. The longitudinal stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place along the section AB.

The longitudinal stress (σ_L) developed in the material

Let, P - Internal pressure of fluid

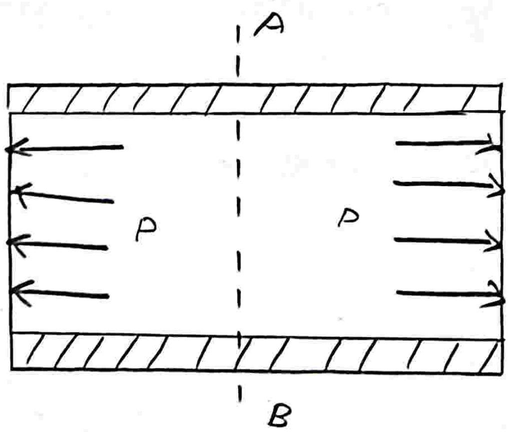
d - Internal diameter of cylinder

t - thickness of cylinder.

The bursting will takes place if the force due to fluid pressure acting on the ends of cylinder is more than resisting force

Force due to fluid pressure } = P x Area on which p is acting

$= P \times \frac{\pi}{4} d^2$



a)

Resisting force = σ_2 x Area on which σ_2 is acting

$= \sigma_2 \times \pi d \times t$

∴ Hence in the limiting case

$$P \times \frac{\pi}{4} \cdot d^2 = \sigma_2 \times \pi d \times t$$

$$\sigma_2 = \frac{P \times \pi/4 \, d^2}{\pi d \times t} = \frac{Pd}{4t}$$

The stress σ_2 is also tensile

Equation (17.2) can be written as,

$$\sigma_2 = \frac{Pd}{2 \times 2t} = \frac{1}{2} \times \sigma_1$$

(or)

longitudinal stress = Half of circumferential stress.

Maximum shear stress:-

At any point in the material of cylindrical shell, there are two principal stresses, namely a stress of magnitude $\sigma_1 = \frac{Pd}{2t}$ acting and a longitudinal stress of magnitude $\sigma_2 = \frac{Pd}{4t}$

∴ Max. shear stress = $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$$= \frac{\frac{Pd}{2t} - \frac{Pd}{4t}}{2}$$

$$= \frac{Pd}{8t}$$

1) A cylindrical pipe of diameter 1.5m and thickness 1.5cm is subjected to an internal fluid pressure of 1.2 N/mm². Determine

i) Longitudinal stress developed in the pipe, and

ii) Circumferential stress developed in the pipe

Solution:

Given:

Dia of pipe, $d = 1.5m$

Thickness, $t = 1.5cm = 1.5 \times 10^{-2} m$

Internal fluid pressure, $P = 1.2 N/mm^2$

As the ratio $\frac{t}{d} = \frac{1.5 \times 10^{-2}}{1.5} = \frac{1}{100}$, which is less than $\frac{1}{20}$, hence this is a case of thin cylinder.

i) The longitudinal stress (σ_2) is given by

$$\sigma_2 = \frac{P \times d}{4t}$$

$$= \frac{1.2 \times 1.5}{4 \times 1.5 \times 10^{-2}} = 30 N/mm^2$$

ii) The circumferential stress (σ_1) is given by

$$\sigma_1 = \frac{Pd}{2t}$$

$$= \frac{1.2 \times 1.5}{2 \times 1.5 \times 10^{-2}}$$

$$= 60 N/mm^2.$$

2) A cylinder of internal diameter 2.5m and thickness 5cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm². Determine the internal pressure of gas.

Given:

Internal dia of cylinder, $d = 2.5m$

Thickness of cylinder, $t = 5cm = 5\times10^{-2}m$

Max. Permissible stress $= 80\ N/mm^2$

As max. permissible stress is given. Hence this should be equal to circumferential stress (σ_1)

$$\sigma_1 = 80\ N/mm^2$$

Let, P = Internal pressure of gas

Using equ (17.1)

$$\sigma_1 = \frac{pd}{2t}$$

$$p = \frac{2t\times\sigma_1}{d} = \frac{2\times5\times10^{-2}\times80}{2.5}$$

$$= 3.2\ N/mm^2.$$

3) A cylinder of internal diameter 0.50m contains air at a pressure of 7 N/mm². If the max permissible stress induced in the material is 80 N/mm², Find the thickness of the cylinder.

Given:

Internal dia of cylinder, $d = 0.50m$

Internal pressure of air, $P = 7\ N/mm^2$

Max. permissible stress in the material means the circumferential stress (σ_1)

$\therefore$ circumferential stress, $\sigma_1 = 80 N/mm^2$

Let, t - Thickness of the cylinder

Using equ (17.1)

$$\sigma_1 = \frac{pd}{2t}$$

$$t = \frac{pd}{2 \times \sigma_1} = \frac{7 \times 0.50}{2 \times 80}$$

$$= 0.021875 m.$$

$$= 2188 cm$$

If the value of t is taken 2.1875 cm, the stress induced will be 80 N/mm2. If the value of t less than 2.1875 cm, the stress induced will be more than 80 N/mm2. But the stress induced should not be more than 80 N/mm2.

Hence, take $t = 2.188 cm$

4) A thin cylinder of internal diameter 1.25 m contains a fluid at an internal pressure of $2 N/mm^2$. Determine the max. thickness of the cylinder.

i) The longitudinal stress is not to exceed $30 N/mm^2$

ii) The circumferential stress is not to exceed 45 N/mm2.

Given:

Internal dia of cylinder, $d = 1.25m$

Internal pressure of fluid $p = 2N/mm^2$

Longitudinal stress, $\sigma_2 = 30\ N/mm^2$

Circumferential stress, $\sigma_1 = 45\ N/mm^2$.

Using equ (17.1),

$$\sigma_1 = \frac{pd}{2t}$$

$$t = \frac{p \times d}{2 \times \sigma_1} = \frac{2 \times 1.25}{2 \times 45} = 0.0277m$$

$$= 2.77cm \quad \text{— } ①$$

Using equ (17.2),

$$\sigma_2 = \frac{pd}{4t}$$

$$t = \frac{pd}{4 \times \sigma_2} = \frac{2 \times 1.25}{4 \times 30} = 0.0208m$$

$$= 2.08cm. \quad \text{— } ②$$

From equ ① ~~&~~ ② it is clear that t should not be less than 2.77cm

Take,
$t = 2.80cm$.

5) A water main 80cm diameter contains under at a pressure head of 100m. If the weight density of water is 9810 N/m^3, find the thickness of metal required for the water main. given the permissible stress as 20 N/mm^2.

Given: Dia of main, $d = 80cm$

Pressure head of water, $h = 100m$

Unit density of water, $w = \rho \times g = 1000 \times 9.81$
$= 9810 N/m^2$

Permissible stress $= 20 N/mm^2$

Permissible stress is equal to circumferential stress (σ_1)

$$\sigma_1 = 20 N/mm^2$$

Pressure of water inside the water main,

$$P = \rho \times g \times h = wh = 9810 \times 100 N/m^2$$

$$P = \frac{9810 \times 100}{1000^2 mm^2} N/mm^2$$

$$= 0.981 N/mm^2$$

Let, t = Thickness of metal required

using equation (17.1)

$$\sigma_1 = \frac{P \times d}{2 \times t}$$

$$t = \frac{P \times d}{2 \times \sigma_1} = \frac{0.981 \times 80}{2 \times 20} = 2cm.$$

Effect of internal pressure on the Dimensions of a thin cylindrical shell:

When a fluid having internal pressure (P) is stored in a thin cylindrical shell, due to internal pressure of fluid the stresses set up at any point of the material of shell are:

i) Hoop or circumferential stress (σ_1) acting on longitudinal section.

ii) Longitudinal stress (σ_2) acting on circumferential section.

Let, P - Internal pressure of fluid

L - Length of cylindrical shell

d - Diameter of cylindrical shell

t - Thickness of cylindrical shell

E - Modulus of elasticity for material of shell.

σ_1 - Hoop stress in material

σ_2 - Longitudinal stress in material

δd - change in diameter due to stress in material

δL - change in Length

δV - change in volume.

The values of σ_1 & σ_2 is given by equ (17.1) & (17.2),

$$\sigma_1 = \frac{Pd}{2t}$$

$$\sigma_2 = \frac{P \times d}{4t}$$

Let, e_1 - circumferential strain

e_2 - Longitudinal strain

Then circumferential strain,

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E}$$

$$= \frac{Pd}{2tE} - \frac{\mu Pd}{4tE}$$

$$= \frac{pd}{2tE}\left(1-\frac{\mu}{2}\right)$$

Longitudinal strain,

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E}$$

$$= \frac{pd}{4tE} - \frac{\mu pd}{2tE}$$

$$= \frac{pd}{2tE}\left(\frac{1}{2}-\mu\right).$$

But circumferential stress is given as,

$$e_1 = \frac{\text{change in circumference due to pressure}}{\text{original circumference}}$$

$$= \frac{\text{final circumference} - \text{original circumference}}{\text{original circumference}}$$

$$= \frac{\pi(d+\delta d) - \pi d}{\pi d}$$

$$= \frac{\pi d + \pi\delta d - \pi d}{\pi d} = \frac{\pi\delta d}{\pi d}$$

$$= \frac{\delta d}{d} \text{ (or) } \left(\frac{\text{change in dia}}{\text{original dia}}\right)$$

Equate two values of e_1 given by equa (17.6) & (17.9), we get,

$$\frac{\delta d}{d} = \frac{pd}{2tE}\left(1-\frac{\mu}{2}\right)$$

change in dia,

$$\delta d = \frac{pd^2}{2tE}\left(1-\frac{\mu}{2}\right)$$

$$e_2 = \frac{\text{change in length due to pressure}}{\text{original length}}$$

$$= \frac{\delta L}{L}$$

Equating two values of e_2 given by equ (17.8) & (17.12)

$$\frac{\delta L}{L} = \frac{pd}{2tE}\left(\frac{1}{2}-\mu\right)$$

∴ change in length,

$$\delta L = \frac{p \times d \times L}{2tE}\left(\frac{1}{2}-\mu\right)$$

Volumetric strain:

It is defined as change in volume divided by original volume.

$$\text{Volumetric strain} = \frac{\delta V}{V}$$

Problems:

1) Calculate i) change in diameter, (ii) change in length and (iii) change in volume of thin cylindrical shell 100cm diameter, 1cm thick and 5m long when subjected to internal pressure 3N/mm². Take the value $E = 2\times10^5$ N/mm² & Poisson's ratio, $\mu = 0.3$

Given:

Dia of shell, $d = 100$cm

Thickness of shell, $t = 1$cm

Length of shell, $L = 5$m $= 5\times100 = 500$cm

Internal pressure, $P = 3$ N/mm²

young's modulus, $E = 2\times10^5$ N/mm²

Poisson's ratio, $\mu = 0.30$

i) change in diameter (δd) is given by equ (17.11) as

$$\delta d = \frac{pd^2}{2tE}\left(1-\frac{\mu}{2}\right)$$

$$= \frac{3\times100^2}{2\times1\times2\times10^5}\left[1-\frac{1}{2}\times0.30\right]$$

$$= \frac{3}{40}(1-0.15) = 0.06375\text{cm}.$$

ii) change in length (δL) is given by equ (17.14)

$$\delta L = \frac{PdL}{2tE}\left(\frac{1}{2}-\mu\right)$$

$$= \frac{3\times100\times500}{2\times1\times2\times10^5}\left(\frac{1}{2}-0.30\right)$$

$$= \frac{15}{40}\times0.20 = 0.075\,cm.$$

iii) change in volume (δV) is given by equ (17.18)

$$\delta V = V\left[2e_1+e_2\right]$$

$$= V\left[2\,\frac{\delta d}{d}+\frac{\delta L}{L}\right]$$

sub the values of δd, δL, d & L, we get

$$\delta V = V\left[2\times\frac{0.06375}{100}+\frac{0.075}{500}\right]$$

$$= V\left[0.001275+0.00015\right] = 0.001425V$$

$$V = \text{original volume} = \frac{\pi}{4}d^2L$$

$$= \frac{\pi}{4}\times100^2\times500\,cm^2 = 3926990.817\,cm^2$$

$$\delta V = 0.001425\times3926990.817$$

$$= 5595.96\,cm^3.$$

2) A cylindrical thin drum 80cm in diameter and 3m long has a shell thickness of 1cm. If the drum is subjected to internal pressure of 2.5 N/mm², determine i) change in diameter ii) change in length iii) change in volume.

given:

$d = 80cm$

$L = 3m = 3\times100 = 300cm$

$t = 1cm$

$P = 2.5\ N/mm^2$

$E = 2\times10^5\ N/mm^2$

$\mu = 0.25$

i) change in diameter (δd) is given by equ (17.11)

$$\delta d = \frac{pd^2}{2tE}\left(1-\frac{1}{2}\times\mu\right)$$

$$= \frac{2.5\times80^2}{2\times1\times2\times10^5}\left(1-\frac{1}{2}\times0.25\right)$$

$$= 0.04(1-0.125) = 0.035\ cm.$$

ii) change in length (δL) is given by equ (17.14)

$$\delta L = \frac{pdL}{2tE}\left(\frac{1}{2}-\mu\right)$$

$$= \frac{2.5\times80\times300}{2\times1\times2\times10^5}\left(\frac{1}{2}-0.25\right) = 0.0375cm$$

iii) using equ (17.15) for volumetric $\left(\frac{\delta V}{V}\right)$, we have

$$\frac{\delta V}{V} = 2\frac{\delta d}{d}+\frac{\delta L}{L}$$

$$= 2\times\frac{0.035}{80}+\frac{0.0375}{300}$$

$$= 0.000875 + 0.000125 = 0.001$$

$$\delta V = 0.001\times V.$$

where, volume $V = \frac{\pi}{4}\times d^2\times L = \frac{\pi}{4}\times80^2\times300$

$$= 1507964.473\ cm^3$$

$\therefore$ change in volume, $\delta V = 0.001\times1507964.473$

$$= 1507.96\ cm^3.$$

8) A cylindrical shell 90cm long 20cm internal diameter having thickness as metal as 8mm is filled with fluid at atmospheric pressure. If additional 20cm3. of fluid is pumped into the cylinder, find i) the pressure exerted by fluid on cylinder ii) The hoop stress induced Take $E = 2\times10^5$ N/mm2 & $\mu = 0.3$.

Given:

$$l = 90cm$$

$$d = 20cm$$

$$t = 8mm \Rightarrow 0.8cm$$

$$= 20cm^3$$

$$V = \frac{\pi}{4} d^2 \times L = \frac{\pi}{4} \times 20^2 \times 90$$

$$= 28274.33 cm^3$$

δV = Volume of additional fluid

$$= 20 cm^3$$

i) Let,

P = pressure exerted by fluid on cylinder

$$E = 2\times10^5 \text{ N/mm}^2$$

$$\mu = 0.3$$

Now using equ (17.16), volumetric strain is

$$\frac{\delta V}{V} = 2e_1 + e_2$$

$$\frac{20}{28274.33} = 2e_1 + e_2$$

$$e_1 = \frac{Pd}{2Et}\left(1-\frac{1}{2}\times\mu\right)$$

$$e_2 = \frac{Pd}{2tE}\left(\frac{1}{2}-\mu\right)$$

sub these values in equ ①, we get

$$\frac{20}{28274.33} = \frac{2pd}{2Et}\left(1-\frac{1}{2}\times\mu\right) + \frac{Pd}{2tE}\left(\frac{1}{2}-\mu\right)$$

$$= \frac{2p\times20}{2\times2\times105\times0.8}\left(1-\frac{1}{2}\times0.3\right) + \frac{P\times20}{0.8\times2\times105}\left(\frac{1}{2}-0.3\right)$$

$$0.000707 = \frac{P}{8000}\times0.85 + \frac{P}{8000}\times0.20 = \frac{1.05P}{8000}$$

$$P = \frac{0.000707\times8000}{1.05} = 5.386\ N/mm^2$$

ii) Hoop stress (σ_1) is given by equ (17.1) as

$$\sigma_1 = \frac{Pd}{2t} = \frac{5.386\times20}{2\times0.8} = 67.33\ N/mm^2.$$

4) A cylindrical vessel whose ends are closed by means of rigid flange plates, is made of steel plate 3mm thick. The length & internal dia of vessel are 50cm & 25cm respectively. Determine the longitudinal and hoop stress in cylindrical shell & volume of vessel. Take $E = 2\times105\ N/mm^2$ & $\mu = 0.3$.

given:

$t = 3mm = 0.3cm$

$L = 50cm$

$d = 25cm$

$P = 3N/mm^2$

$E = 2 \times 10^5 \ N/mm^2$

$\mu = 0.3$

Let, σ_1 = Hoop stress

σ_2 = Longitudinal stress

Using equ (17.1) for hoop stress,

$$\sigma_1 = \frac{p \times d}{2t} = \frac{3 \times 25}{2 \times 0.3} = 125 \ N/mm^2$$

Using equ (17.2) for longitudinal stress,

$$\sigma_2 = \frac{p \times d}{4t} = \frac{3 \times 25}{4 \times 0.3} = 62.5 \ N/mm^2$$

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu \times \sigma_2}{E}$$

$$= \frac{1}{E}(\sigma_1 - \mu \times \sigma_2)$$

$$= \frac{1}{2 \times 10^5}(125 - 62.5 \times 0.3)$$

$$= \frac{1}{2 \times 10^5}(125 - 18.75) = \frac{106.25}{2 \times 10^5}$$

$$= 53.125 \times 10^{-5}$$

But circumferential strain is given by equ (17.9)

$$e_1 = \frac{\delta d}{d}$$

Equating the two values of circumferential strain e_1, we get

$$\frac{\delta d}{d} = 53.125 \times 10^{-5}$$

$$\delta d = 53.125 \times 10^{-5} \times d = 53.125 \times 10^{-5} \times 25$$

$$= 0.0133 cm$$

$\therefore$ Increase in diameter, $\delta d = 0.0133 cm$

$$e_2 = \frac{\delta L}{L} = \frac{\sigma_2}{E} - \frac{\mu \times \sigma_1}{E}$$

$$= \frac{1}{E}(\sigma_2 - \mu \times \sigma_1)$$

$$e_2 = \frac{\delta L}{L} = \frac{\sigma_2}{E} - \frac{\mu \times \sigma_1}{E}$$

$$= \frac{1}{E}(\sigma_2 - \mu \times \sigma_1)$$

$$= \frac{1}{2\times10^5}(62.5 - 125\times0.3) = \frac{1}{2\times10^5}(62.5-37.5)$$

$$= \frac{25}{2\times10^5} = 12.5\times10^{-5}$$

$$\delta L = 12.5\times10^{-5}\times L$$

$$= 12.5\times10^{-5}\times50 = 0.00625 cm$$

Volumetric strain is given by equ (17.16) as

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta L}{L}$$

$$= 2e_1 + e_2 = 2\times53.125\times10^{-5} + 12.5\times10^{-5}$$

$$= 106.25\times10^{-5} + 12.5\times10^{-5} = 118.75\times10^{-5}$$

Increase in volume,

$$\delta V = 118.75\times10^{-5}\times V$$

$$= 118.75\times10^{-5}\times\frac{\pi}{4}\times25^2\times50$$

$$= 29.13 cm^2.$$

5) A cylindrical vessel is 1.5m diameter and 4m long is closed at ends by rigid plates. It is subjected to an internal pressure of $3 N/mm^2$. If the max principal stress not to exceed $150 N/mm^2$. find the thickness of shell. Assume $E = 2\times10^5 N/mm^2$ & poisson's ratio $= 0.25$. Find the change in diameter, length & volume of shell.

given: $d = 1.5m = 1500mm$

$L = 4m = 4000mm$

$P = 3N/mm^2$

Max. principle stress $=150\ N/mm^2$

$\therefore$ circumferential stress $\sigma_1 = 150\ N/mm^2$

value of $E = 2\times10^5\ N/mm^2$

$\mu = 0.25$

Let, t - thickness of shell

δd - change in dia

δL - change in length

i) $\sigma_1 = \dfrac{P\times d}{2t}$

$t = \dfrac{P\times d}{2\times\sigma_1} = \dfrac{3\times1500}{2\times150}$

$= 15mm$

ii) $\delta d = \dfrac{Pd^2}{2t\times E}\left(1-\frac{1}{2}\times\mu\right)$

$= \dfrac{3\times1500^2}{2\times15\times2\times10^5}\left(1-\frac{1}{2}\times0.25\right)$

$= 0.984mm$

iii) $\delta L = \dfrac{P\times d\times L}{2t\times E}\left(\frac{1}{2}-\mu\right)$

$= \dfrac{3\times1500\times4000}{2\times15\times2\times10^5}\left(\frac{1}{2}-0.25\right)$

$= 0.75mm$

iv) $\dfrac{\delta V}{V} = \dfrac{P\times d}{2E\times t}\left(\frac{5}{2}-2\times\mu\right)$

$= \dfrac{3\times1500}{2\times2\times10^5\times15}\left(\frac{5}{2}-2\times0.25\right) = \dfrac{3\times1500\times2}{4\times10^5\times15}$

$\delta V = \dfrac{3}{2000}\times V = \dfrac{3}{2000}\times\left(\frac{\pi}{4}\times d^2\times L\right)$

$= \dfrac{3}{2000}\times\left(\frac{\pi}{4}\times1500^2\times4000\right)$

$= 10602875mm^2.$

UNIT- IV

PRINCIPAL STRESS AND THEORIES OF ELASTIC FAILURE

Introduction :

When some external load is applied on a body, the stresses and strains are Produced in the body. The stresses and directly Proportional to the strain within the elastic limit. This means when the load is removed. The body will return to its original shape. There is no Permanent deformation in the body.

However, if the stress Produced in the body due to the application of the load is beyond the elastic limit, the Permanent deformation occur in the body. This means if the load is removed. whenever Permanent deformations occurs in the body, the body is said to have "failed". This should be clear that failure does not mean rupture of the body.

Let us the consider the failure of a bar in a Simple tensile test. The tensile stress is directly Proportional to the tensile strain upto elastic limit. This means that there is a definite value of tensile stress upto elastic limit. Beyond the elastic limit if the tensile stress increases, But the failure of the Bar will take place. certain theories have advanced to explain the cause of failure. According to the important theories, the failure takes place when a certain Limiting value is reached by one of following :

1) The maximum Principal stress
2) The maximum Principal strain
3) The maximum shear stress
4) The maximum strain energy
5) The maximum shear strain energy

$\sigma_1, \sigma_2, \sigma_3$ = Principal stresses in any Complex system

σ^* = tensile or compressive stress at the elastic limit.

1) Maximum Principal stress theory :

According to this theory, the failure of a material will occur when the maximum Principal tensile stress (σ_1) in the complex system reaches the value of the maximum stress at the elastic Limit in simple tension or the maximum Principal stress. (The maximum Principal compressive stress reaches the value of the maximum stress at the elastic limit in simple compression.

Let in a complex three dimensional stress system.

σ_1, σ_2 and σ_3 = Principal stress at a point in three Perpendicular directions. The stresses σ_1 and σ_2 are tensile and σ_3 is compressive. Also σ_1 is more than σ_2.

σ_t^* = tensile stress at elastic limit in simple tension.

σ_c^* = compressive stress at elastic limit in simple compression.

Then according to this theory, the failure will take place if.

$\sigma_1 \geq \sigma_t^*$ in simple tension

or $|\sigma_3| \geq \sigma_c^*$ in simple compression

where $|\sigma_3|$ represents the absolute value of σ_3.

This is the simplest and oldest theory of failure and is known as Rankine's theory. If the maximum principal stress (σ_1) is the design criterion, then maximum principal stress must not exceed the permissible stress (σ_1) for the given material.

Hence $\sigma_1 = \sigma_t$

σ_t = Permissible stress and is given by

$\sigma_t = \sigma_t^* / \text{safety factor}$

Problem 1 :

The Principal stresses at a point in an elastic material are $100 N/mm^2$ (tensile), $80 N/mm^2$ (tensile) and $50 N/mm^2$ (compressive). If the stress at the elastic limit in simple tension is $200 N/mm^2$, determine whether the failure of material will occur according to maximum Principal stress theory. If not, then determine the factor of safety.

Given :

$\sigma_1 = 100 N/mm^2$ (tensile)

$\sigma_2 = 80 N/mm^2$ (tensile)

$\sigma_3 = 50 N/mm^2$ (compressive) $= -50 N/mm^2$

stress at elastic limit in simple tension,

$\sigma_t^* = 200 N/mm^2$

i) To determine the whether failure of material will occur or not :

From the three given stresses, the maximum Principal tensile stress is $\sigma_1 = 100 N/mm^2$. and the stress at elastic limit in simple tension is

$\sigma_t^* = 200 \, N/mm^2$. As σ_t is less than σ_t^*, the failure will not occur according to maximum Principal stress theory.

ii) Factor of safety:

$$\sigma_1 = \sigma_t$$

$$\sigma_t = 100 \, N/mm^2$$

$$\sigma_t = \frac{\sigma_t^*}{FOS}$$

$$FOS = \frac{\sigma_t^*}{\sigma_t}$$

$$= \frac{100}{200} = 2.0.$$

Maximum Principal strain theory:

This theory is due to saint venant. According to this theory, the failure will occur in a material when the maximum Principal strain reaches the strain due to yield stress in simple tension or when the maximum Principal strain Reaches the strain due to yield stress in simple compression. Yield stress is the maximum stress at elastic

Limit. consider a three dimensional stress system.

Principal strain in the direction of Principal stress σ_1 is,

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E}$$

$$= \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2 + \sigma_3)\right]$$

Principal strain in the direction of Principal stress σ_3 is

$$e_3 = \frac{1}{E}\left[\sigma_3 - \mu(\sigma_1 + \sigma_2)\right]$$

strain due to yield stress in simple tension.

$= \frac{1}{E} \times$ yield stress in tension

$$= \frac{1}{E} \times \sigma_t^*$$

strain due to yield stress in simple compression $= \frac{1}{E} \times \sigma_c^*$

where yield stress is the maximum stress at elastic limit.

According to this theory, the failure of the material will taken place when

$$e_1 \geq \frac{\sigma_t^*}{E}$$

$$|e_3| \geq \frac{\sigma_c^*}{E}$$

substituting the values of e_1 and e_3, we get the conditions of failure as:

i) $\frac{1}{E}[\sigma_1 - \mu(\sigma_2+\sigma_3)] \geq \frac{1}{E} \times \sigma_t^*$

$$\sigma_1 - \mu(\sigma_2+\sigma_3) \geq \sigma_t^*$$

ii) $\left|\frac{1}{E}[\sigma_3 - \mu(\sigma_1+\sigma_2)]\right| \geq \frac{1}{E} \times \sigma_c^*$

$$|[\sigma_3 - \mu(\sigma_1+\sigma_2)]| \geq \sigma_c^*$$

For actual design instead of σ_t^* or σ_c^*, the permissible stress in simple tension or compression should be used where

$$\sigma_t = \frac{\sigma_t^*}{\text{safety factor}}$$

$$\sigma_c = \frac{\sigma_c^*}{\text{safety factor}}$$

Hence for design purpose,

$$\sigma_1 - \mu(\sigma_2+\sigma_3) = \sigma_t$$

$$|[\sigma_3 - \mu(\sigma_1+\sigma_2)]| = \sigma_c$$

8) The Principal stresses at a point in an elastic material are $200 N/mm^2$ (tensile), $100 N/mm^2$ (tensile), $50 N/mm^2$ (compressive). If the stress at the elastic Limit in simple tension is $200 N/mm^2$, determine whether the failure of material will occur according to maximum Principal strain theory. Take poisson's ratio $= 0.3$.

Given:

$\sigma_1 = 200 N/mm^2$ (tensile)

$\sigma_2 = 100 N/mm^2$ (tensile)

$\sigma_3 = 50 N/mm^2$ (compressive) $= -50 N/mm^2$

stress at elastic Limit in simple tension,

$\sigma_t^* = 200 N/mm^2$

Poisson's ratio; $\mu = 0.3$

To determine whether failure of material will occur or not according to maximum Principal strain theory.

$$e_1 = \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2 + \sigma_3)\right]$$

$$= \frac{1}{E}\left[200 - 0.3\left[100 + (-50)\right]\right]$$

$$= \frac{1}{E}[200 - 30 + 15] = \frac{185}{E}$$

strain due to stress at elastic limit in simple tension,

$$e_t^* = \frac{\sigma_t^*}{E} = \frac{200}{E}$$

According to maximum Principal strain theory; the failure of a material occurs if the maximum Principal strain reaches the strain due to stress at elastic limit in simple tension.

Here $e_1 < e_t^*$, hence failure will not occur.

3) Determine the Diameter of a bolt which is subjected to an axial pull of 9KN together with a transverse shear force of 4.5KN using: i) Maximum Principal stress theory ii) Maximum Principal strain theory.

Given the elastic limit in tension $= 225 N/mm^2$, fos $= 3$ poisson's ratio $= 0.3$.

Soln:

Axial pull, $P = 9KN = 9 \times 1000N = 9000N$

Transverse shear Force, $F = 4.5KN = 4500N$

Elastic limit in tension, $\sigma_t^* = 225 N/mm^2$

FOS $= 3$, $\mu = 0.3$

The Permissible stress in tension is given by equation,

$$\sigma_t = \frac{\sigma_t^*}{\text{safety factor}} = \frac{225}{3} = 75 N/mm^2$$

d = diameter of bolt in mm.

$$\text{tensile stress}, \sigma = \frac{P}{\text{Area of cross Section}} = \frac{P}{\pi/4\, d^2} = \frac{4P}{\pi d^2}$$

$$= \frac{4 \times 9000}{\pi d^2} = \frac{11459}{d^2} N/mm^2$$

$$\text{shear stress}, \tau = \frac{F}{\pi/4\, d^2} = \frac{4F}{\pi d^2} = \frac{4 \times 4500}{\pi d^2}$$

$$= \frac{5729.5}{d^2} N/mm^2$$

calculate the maximum and minimum Principal stresses σ_1 and σ_2.

$$\sigma_1 \text{ and } \sigma_2 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{11459}{2 \times d^2} \pm \sqrt{\left(\frac{11459}{2 \times d^2}\right)^2 + \left(\frac{5729.5}{d^2}\right)^2}$$

$$= \frac{5729.5}{d^2} \pm \sqrt{\left(\frac{5729.5}{d^2}\right)^2 + \left(\frac{5729.5}{d^2}\right)^2}$$

$$= \frac{5729.5}{d^2} \pm \frac{5729.5}{d^2}\sqrt{(1+1)}$$

$$= \frac{5729.5}{d^2} \pm \frac{8103}{d^2}$$

$$\sigma_1 = \frac{5729.5}{d^2} + \frac{8103}{d^2} = \frac{13832.5}{d^2} \ N/mm^2$$

$$\sigma_2 = \frac{5729.5}{d^2} - \frac{8103}{d^2} = \frac{-2373.5}{d^2} \ N/mm^2$$

Hence the Principal stresses in the bolt are :

σ_1, σ_2 and 0 or $\frac{13832.5}{d^2}$, $\frac{-2373.5}{d^2}$ and 0.

i) Diameter of bolt according to maximum Principal stress theory :

$$\sigma_1 = \sigma_t$$

$$\frac{13832.5}{d^2} = 75$$

$$d^2 = \frac{13832.5}{75}$$

$$d = \sqrt{\frac{13832.5}{75}} = 13.58 = 13.6 \, mm$$

ii) Diameter of bolt according to maximum principal strain theory The three Principal stresses are :

$$\sigma_1 = \frac{13832.5}{d^2} \ N/mm^2$$

$$\sigma_2 = \frac{-2373.5}{d^2} \ N/mm^2 \quad \text{and} \quad \sigma_3 = 0$$

$$\text{Maximum strain} = \frac{\sigma_1}{E} - \frac{\mu}{E}[\sigma_2 + \sigma_3]$$

$$= \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$$

$$= \frac{1}{E}\left[\frac{13832.5}{d^2} - 0.3\left(\frac{-2373.5}{d^2} + 0\right)\right]$$

$$= \frac{1}{E}\left[\frac{13832.5}{d^2} + \frac{712.05}{d^2}\right]$$

$$= \frac{1}{E} \times \left[\frac{14544.55}{d^2}\right]$$

Maximum strain due to Permissible stress in

$$\text{tension} = \frac{1}{E} \times \sigma_t$$

$$= \frac{1}{E} \times 75$$

Hence equating the two values,

$$\frac{1}{E} \times \frac{14544.55}{d^2} = \frac{1}{E} \times 75$$

$$d = \sqrt{\frac{14544.55}{75}} = 13.92 \text{ mm}$$

Maximum shear stress theory :

This theory is due to Guest and Tresca and therefore known as Guest's theory. According to this theory, the failure of a material will occur when the maximum shear stress in a material reaches the value of maximum shear stress in simple tension at the elastic limit. The maximum shear stress in the material is equal to half the difference b/w maximum and minimum principal stress.

Max. shear stress in the material = Half of difference of maximum and minimum Principal stresses

$$= \frac{1}{2}[\sigma_1 - \sigma_3]$$

In case of simple tension, at the elastic limit the Principal stresses are σ_t^* 0, 0

Max. shear stress in simple tension at elastic limit = Half of the difference of maximum and minimum principal stresses

$$= \frac{1}{2}[\sigma_t^* - 0] = \frac{1}{2}\sigma_t^*$$

For the failure of material,

$$\frac{1}{2}[\sigma_1 - \sigma_3] \geq \frac{1}{2}\sigma_t^* \quad \text{or} \quad (\sigma_1 - \sigma_3) \geq \sigma_t^*$$

$$\sigma_t = \frac{\sigma_t^*}{\text{safety factor}}$$

Hence the design, the following equation should be used.

$(\sigma_1 - \sigma_3) = \sigma_t$

Equation is to be used ~~be~~ for design purpose only. It should not be used for determining the failure of the material due to maximum shear stress theory.

4) For the data given Previous Problem. determine whether failure of material will occur or not according to maximum shear stress theory.

Soln:

$\sigma_1 = 200 N/mm^2$ (tensile)

$\sigma_2 = 100 N/mm^2$ (tensile)

$\sigma_3 = 50 N/mm^2$ (compressive) $= -50 N/mm^2$

$\sigma_t^* = 200 N/mm^2$

Max. shear stress developed in the material = Half of diff of maximum and minimum principal stresses

$$= \frac{1}{2}[\sigma_1 - \sigma_3] = \frac{1}{2}[200 - (-50)]$$

$$= \frac{250}{2} = 125 N/mm^2$$

Hence σ_1 is maximum and σ_3 is minimum.

Max. shear stress at elastic limit in tension

$= \frac{1}{2} \times \sigma_t^*$

$= \frac{1}{2} \times 200 = 100 N/mm^2.$

As maximum shear stress developed in the material is $125 N/mm^2$.

5) At a Section of a mild steel shaft, the maximum torque is 8437.5 Nm and maximum Bending moment is 5062.5 Nm. The diameter of shaft is 90mm and the stress at the elastic Limit in simple tension for the material of the shaft is $220 N/mm^2$. Determine whether the failure of the material will occur or not according to maximum shear stress theory. If not, then find the factor of safety.

Given:

Maximum Torque, $T = 8437.5 Nm$

Maximum Bending moment, $M = 5062.5 Nm$

stress at elastic limit in simple tension,

$\sigma_t^* = 220 N/mm^2 = 220 \times 10^6 N/m^2 = 220 MN/m^2$

Dia of shaft, $d = 90mm = 0.09m$.

$$T = \frac{\pi}{16} \times d^3 \times \tau$$

$$\tau = \frac{16 \times T}{\pi d^3} = \frac{16 \times 8437.5}{\pi \times (0.09)^3} N/m^2 = 58.946 \times 10^6 N/m^2$$

$$= 58.946 \, MN/m^2$$

Also we know that $\frac{M}{I} = \frac{\sigma_b}{y}$

$$\sigma_b = \frac{M \times Y}{I} \qquad Y = \frac{d}{2}$$

$$= \frac{M \times d}{I \times 2} = \frac{M \times d}{\left(\frac{\pi}{64} d^4\right) \times 2}$$

$$= \frac{32M}{\pi d^3} = \frac{32 \times 5062.5}{\pi \times (0.09)^3}$$

$$= 70.735 \times 10^6 N/m^2 = 70.735 \, MN/m^2.$$

on the surface of the shaft, at any point the shear stress is $58.946 MN/m^2$ and Bending stress is $70.735 MN/m^2$.

$$\sigma_1 \text{ and } \sigma_2 = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{70.735}{2} \pm \sqrt{\left(\frac{70.735}{2}\right)^2 + (58.946)^2}$$

$$= 35.365 \pm 68.75$$

$$= 104.115 \, MN/m^2 \text{ and } -33.385 \, MN/m^2$$

$\therefore \sigma_1 = 104.115 MN/m^2$ and $\sigma_2 = -33.385 MN/m^2$

apply the maximum shear stress theory.

Maximum shear stress due to principal stresses = Half of difference b/w maximum and minimum Principal stresses

$$= \frac{1}{2}[104.115 - (-33.385)]$$

$$= \frac{1}{2}[104.115 + 33.385]$$

$$= 68.75 MN/m^2$$

Maximum shear stress in simple tension

$$= \frac{1}{2}[\sigma_t^* - 0] = \frac{\sigma_t^*}{2}$$

$$= \frac{220}{2} = 110 MN/m^2$$

Factor of safety:

Let σ_t = Allowable tensile stress in simple tension.

Then principal stresses in simple tension will be $\sigma_t, 0, 0$

the maximum allowable shear stress in simple tension will be

$$= \frac{1}{2}[\sigma_t - 0] = \frac{\sigma_t}{2}$$

Equating the two equations,

$$68.75 = \frac{\sigma_t}{2}$$

$$\sigma_t = 68.75 \times 2 = 137.5\, MN/m^2$$

$$FOS = \frac{\sigma_t^*}{\sigma_t} = \frac{220}{137.5} = 1.6$$

Maximum strain energy theory :

This theory is due to Haigh and is known as Haigh's theory. According to this theory, the failure of a material occurs when the total strain energy per unit volume in the material reaches the strain energy per unit volume of the material at the elastic limit in simple tension.

We have stated that the strain energy in a body is equal to work done by the load (P) in straining the material and it is equal to $\frac{1}{2} \times P \times \delta L$

For the failure of the material

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \geq \frac{1}{2E} \times (\sigma_t^*)^2$$

$$\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \geq (\sigma_t^*)^2$$

For a two-dimensional stress system, $\sigma_3 = 0$.

Hence above equation become as

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2) \geq (\sigma_t^*)^2$$

For actual design in stead of σ_t^*, the allowable stress in simple tension should be considered where

$$\sigma_t = \frac{\sigma_t^*}{\text{Factor of safety}}$$

Hence for design, the following equation should be used

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 \times \sigma_2) = \sigma_t^2$$

6) For the data given in Previous problem. It is not used for determining the failure of material will occur or not according to maximum strain energy theory.

<u>soln:</u>

$\sigma_1 = 200 N/mm^2$ (tensile)

$\sigma_2 = 100 N/mm^2$ (tensile)

$\sigma_3 = 50 N/mm^2$ (compressive) $= -50 N/mm^2$

$\mu = 0.3$

Elastic limit in simple tension, $\sigma_t^* = 200 N/mm^2$

Total strain energy Per unit volume in the material

$$= \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right]$$

$$= \frac{1}{2E}\left[200^2 + 100^2 + (-50)^2 - 2\times0.3\left[200\times100 + 100\times(-50) + (-50)\times200\right]\right]$$

$$= \frac{1}{2E}\left[40000 + 10000 + 2500 - 0.6(20000 - 5000 - 10000)\right]$$

$$= \frac{1}{2E}\left[52500 - 0.6\times5000\right] = \frac{1}{2E}\left[49500\right]$$

strain energy Per unit volume at elastic limit in simple tension

$$= \frac{1}{2E}\times\sigma_t^{*2} = \frac{1}{2E}\times200^2$$

$$= \frac{40000}{2E}$$

Now apply the theory of maximum strain energy.

7) For the data given Previous Problem. determine the diameter of the bolt according to maximum strain energy theory.

Soln:

$P = 9KN = 9000N$

shear force, $F = 4500N$.

$\sigma_t^* = 225 N/mm^2$

safety factor = 3

$\mu = 0.3$

Permissible simple stress in tension,

$$\sigma_t = \frac{\sigma_t^*}{\text{safety factor}} = \frac{225}{3} = 75 N/mm^2$$

$$\sigma = \frac{11459}{d^2} N/mm^2, \quad \tau = \frac{5729.5}{d^2} N/mm^2$$

The maximum and minimum Principal stresses are:

$$\sigma_1 \text{ and } \sigma_2 = \frac{1}{2} \times \sigma \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{11459}{2d^2} \pm \frac{8103}{d^2}$$

$$= \frac{5729.5}{d^2} \pm \frac{8103}{d^2}$$

$$\sigma_1 = \frac{5729.5}{d^2} + \frac{8103}{d^2} = \frac{13832.5}{d^2} N/mm^2$$

$$\sigma_2 = \frac{5729.5}{d^2} - \frac{8103}{d^2} = \frac{-2373.5}{d^2} N/mm^2$$

Hence the principal stresses at the point are:

σ_1, σ_2, and 0.

according to maximum strain energy theory,

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 \times \sigma_2) = \sigma_t$$

$$\left[\frac{13832.5}{d^2}\right]^2 + \left[\frac{-2373.5}{d^2}\right]^2 - 2 \times 0.3 \left[\frac{13832.5}{d^2} \times \frac{(-2373.5)}{d^2}\right]$$

$$= 75^2 \quad (\sigma_t = 75)$$

$$\frac{19134 \times 10^4}{d^4} + \frac{563.35 \times 10^4}{d^4} + \frac{1969.9 \times 10^4}{d^4} = 5625$$

$$\frac{21667.25 \times 10^4}{d^4} = 5625$$

$$d = \left(\frac{21667.25 \times 10^4}{5625}\right)^{1/4} = 10 \times (3.852)^{1/4}$$

$$= 10 \times 1.401 = 14.01 \text{mm}.$$

Maximum shear strain energy theory:

This theory is due to mises and Henky and is Known as Mises - Henky theory. This theory is also called the energy of distortion theory. According to this theory, the failure of material occurs when the total shear strain energy per unit volume in the stressed material reaches a value equal to the shear strain energy per unit volume at the elastic limit in the simple tension test.

The total shear strain energy* per unit volume due to principal stresses σ_1, σ_2 and σ_3 in a stressed

stressed material is given as

$$= \frac{1}{12C}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

The simple tension test is a uniaxial stress system which means the principal stresses are $\sigma_1, 0, 0$.

At the elastic limit the tensile stress in simple test is σ_t^*.

Hence at the elastic limit in simple tension, the principal stresses are $\sigma_t^*, 0, 0$.

The shear strain energy per unit volume at the elastic limit in simple tension will be

$$= \frac{1}{12C}\left[(\sigma_t^*-0)^2+(0-0)^2+(0-\sigma_t^*)^2\right]$$

$$= \frac{1}{12C}\left[2\times \sigma_t^{*2}\right]$$

For the failure of the material,

$$\frac{1}{12C}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \geq \frac{1}{12C}\left[2\times(\sigma_t^*)^2\right]$$

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 \geq 2\times(\sigma_t^*)^2$$

$$\sigma_t = \frac{\sigma_t^*}{\text{safety factor}}$$

Hence for design purpose, the following equation should be used :

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2 \times (\sigma_t)^2$$

For a two-dimensional stress, $\sigma_3 = 0$.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2)^2 + (-\sigma_1)^2 = 2 \times (\sigma_t)^2$$

$$\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 + \sigma_2^2 + \sigma_1^2 = 2 \times \sigma_t^2$$

$$2(\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2) = 2 \times \sigma_t^2$$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_t^2$$

8) For the data is given in Previous Problem. determine whether the failure of the material will occur or not according to maximum shear strain energy.

Soln:

$\sigma_1 = 200 N/mm^2$ (tensile)

$\sigma_2 = 100 N/mm^2$ (tensile)

$\sigma_3 = 50 N/mm^2$ (compressive) $= -50 N/mm^2$

Elastic limit in simple tension ; $\sigma_t^* = 200 N/mm^2$.

$\mu = 0.3$

The total shear strain energy Per unit volume due to principal stresses σ_1, σ_2 and σ_3 in the stressed material.

$$= \frac{1}{12C}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

$$= \frac{1}{12C}\left[(200-100)^2 + (100-(-50))^2 + [-50-200)^2\right]$$

$$= \frac{1}{12C}\left[10000 + 22500 + 62500\right]$$

$$= \frac{1}{12C} \times 95000$$

The shear strain energy per unit volume at elastic limit in simple tension.

$$= \frac{1}{12C}\left[(\sigma_t^* - 0)^2 + (0-0)^2 + (0 - \sigma_t^*)^2\right]$$

$$= \frac{1}{12C} \times 2 \times (\sigma_t^*)^2$$

$$= \frac{1}{12C} \times 2 \times (200^2) = \frac{1}{12C} \times 80000.$$

apply the theory of maximum shear strain energy.

9) For the data given in the previous problem. determine the diameter of the bolt according to maximum shear strain energy.

soln:

$P = 9000 N$

$F = 4500 N$

$\sigma_t^* = 225 N/mm^2$

Safety factor $= 3$, $\mu = 0.3$

Allowable stress in simple tension $= \dfrac{\sigma_t^*}{fos}$

$$\sigma_t = \frac{225}{3} = 75\,N/mm^2$$

$$\sigma = \frac{11459}{d^2}\,N/mm^2,\quad \tau = \frac{5729.5}{d^2}\,N/mm^2$$

The maximum and minimum Principal stresses are

$$\sigma_1 = \frac{13832.5}{d^2}\,N/mm^2 \text{ and } \sigma_2 = \frac{-2373.5}{d^2}\,N/mm^2$$

The third Principal stress, $\sigma_3 = 0$

In this Problem, diameter is to be calculated according to the theory of maximum shear strain energy.

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 = 2\times(\sigma_t)^2$$

$$\left[\frac{13832.5}{d^2} - \left(\frac{-2373.5}{d^2}\right)\right]^2 + \left[\left(\frac{-2373.5}{d^2} - 0\right)\right]^2 + \left[0 - \frac{13832.5}{d^2}\right]^2 = 2\times75^2$$

$$\left[\frac{13832.5+2373.5}{d^2}\right]^2 + \left[\frac{2373.5}{d^2}\right]^2 + \left[\frac{-13832.5}{d^2}\right]^2 = 11250$$

$$\frac{26263\times10^4}{d^4} + \frac{563.35\times10^4}{d^4} + \frac{19134\times10^4}{d^4} = 11250$$

$$\frac{45960.35\times10^4}{d^4} = 11250$$

$$d^4 = \frac{45960.35\times10^4}{11250} = 4.08536\times10^4$$

$$d = [4.08536\times10^4]^{1/4} = (4.08536)^{1/4}\times10$$

$$= 1.4217\times10 = 14.217\,mm$$

UNIT-5

ANALYSIS OF PLANE MEMBERS

INTRODUCTION:

A Structure made up of several bars (or members) riveted or welded together is known as frame. If the frame is composed of such members which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external load, then the frame is known as perfect frame. Though in actual practice the members are welded or riveted together at their joints, yet for calculation purposes the joints are assumed to be hinged or pin-joined. In this chapter, we shall discuss how to determine the forces in the members of perfect frame, when it is subject to same external load.

Types of Frames:

The different types of frames are :

i) perfect frame , and

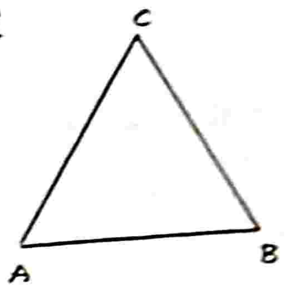
ii) Imperfect frame

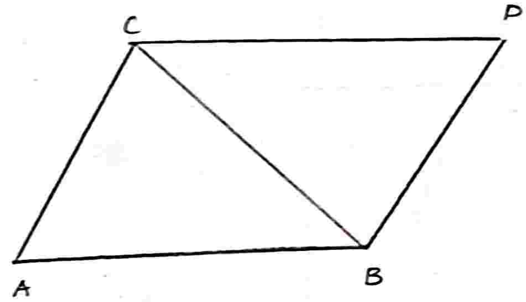
Imperfect frame may be a deficient frame or a redundant frame.

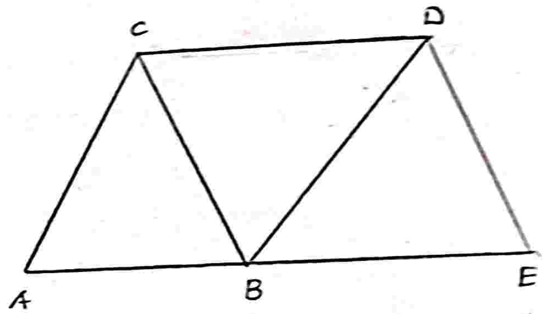
Perfect frame:

The frame which is composed of such members, which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external load, is known as perfect frame. The simplest perfect frame is a triangle as shown in Fig. which consists three members and three joints. The three members are AB, Bc and Ac Whereas the three joints. The three members are A, B and c. This frame can be easily analysed by the condition of equilibrium.

let the two members CD and BD and a joint D are added to the triangle frame ABC. Now, we get a frame ABCD as shown in Fig. This frame can also be analysed by the conditions of equilibrium. This frame is also known as perfect frame.







Assumptions Made in Finding out the Forces in a Frame:

The assumptions made in finding out the forces in a frame are:

i) The frame is a perfect frame

ii) The frame carries load at the joint

iii) All the members are pin-jointed.

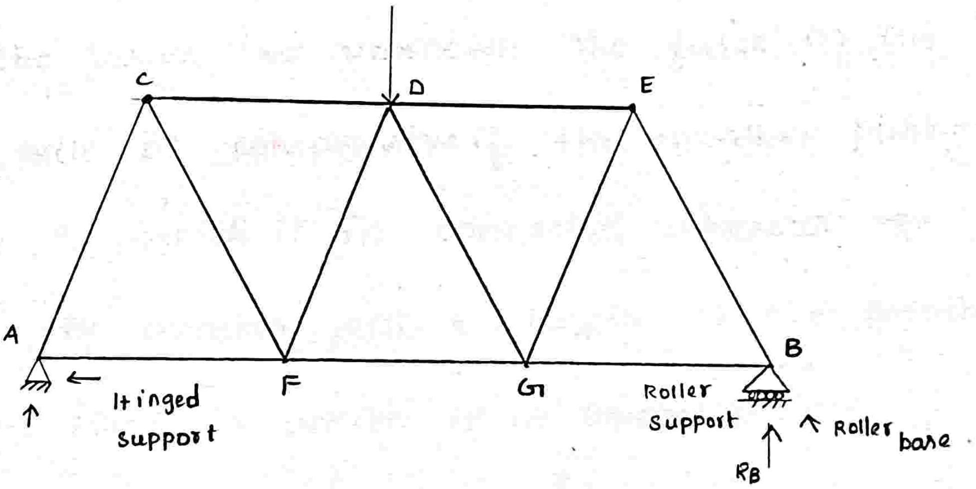
Reactions of supports of a frame:

The frames are generally supported.

i) on roller support or

ii) on a hinged support

If the frame is supported on a roller support then line of action of the reaction will be at right angles to the roller base as shown in Fig



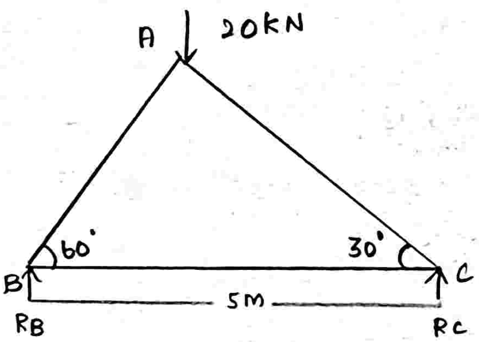
If the frame is supported on hinged support, then the Line of action of the reaction depend upon the load system on the frame.

Method of Joints:

In this method, after determining the reactions at the supports, the equilibrium of every joint is considered. This means the sum of all the vertical forces as well as the horizontal forces acting on a joint is equated to zero. The joint should be selected such a way that at any time there are only two members, in which the forces are unknown. The force in the member will be compressive if the member pushes the joint to which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.

Problem 1

Find the forces in the members AB, AC and BC of the truss shown in Fig.



Soln :

First determine the reactions RB and Rc. The line of action of load of 20 kN acting at A is vertical. This load is at a distance of AB × cos 60° from the point B. Now let us find the distance AB.

The triangle ABC is a right angled triangle with angle BAC = 90°. Hence AB will be equal to BC × cos 60°.

$$AB = 5 \times \cos 60^\circ = 5 \times \frac{1}{2} = 2.5\,m$$

Now the distance of line of action of 20 kN from B is

$$AB \times \cos 60^\circ \text{ or } 2.5 \times \frac{1}{2} = 1.25\,m.$$

Taking the moment about B, we get

$$R_C \times 5 = 20 \times 1.25$$

$$= 25$$

$$R_C = \frac{25}{5}$$

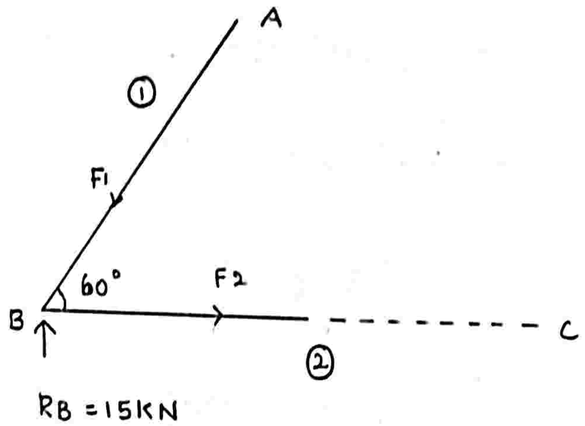
$$= 5\,kN$$

$$R_B = \text{Total load} - R_C$$

$$= 20 - 5 = 15\,kN$$

Now let us consider the equilibrium of the various joints

Joint B



let F_1 = Force in member AB

F_2 = Force in member BC

let the force F_1 is acting toward the joint B and the force F_2 is acting away from the joint B as shown in Fig. (The reaction R_B is acting vertically up. The force F_2 is horizontal. The reaction R_B will be balanced R_B. Hence F_1 must act towards the joint B so that its vertical component of F_1. The vertical component of F_1

must act downwards to balance R_B. Hence F_1 must act towards the joint B so that its vertical component is downward. Now the horizontal component of F_1 is toward the joint B. Hence force F_2 must act away from the joint to balanced the horizontal component of F_1).

Resolving the forces acting on the joint B, vertically.

$$F_1 \sin 60° = 15$$

$$F_1 = \frac{15}{\sin 60°} = \frac{15}{0.866}$$

$$= 17.32 \text{ KN (compressive)}.$$

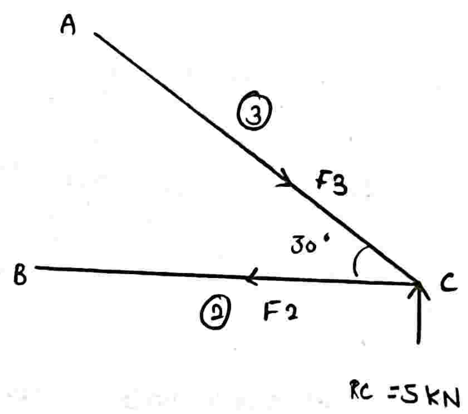
As F_1 is pushing the joint B, hence this Force will be compressive. Now resolving the forces horizontally, we get.

$$F_2 = F_1 \cos 60° = 17.32 \times \frac{1}{2}$$

$$= 8.66 \text{ KN (tensile)}$$

As F_2 is pulling the joint B, hence this force will be tensile.

Joint C



let

F_3 = Force in member AC

F_2 = Force in the member BC

The force F_2 has already been calculated in magnitude and direction. We have seen that force F_2 is tensile and hence it will pull the joint C. Hence it must act away from the joint C as shown in Fig.

Resolving forces vertically, we get

$$F_3 \sin 30° = 5 kN$$

$$F_3 = \frac{5}{\sin 30°}$$

$$= 10 kN \text{ (compressive)}$$

As the force F_3 is pushing the joint c, hence it will be compressive.

Problem 2

A truss of span 7·5m carries a point load of 1kN at joint D as shown in Fig. Find the reactions and forces in the members of the truss.

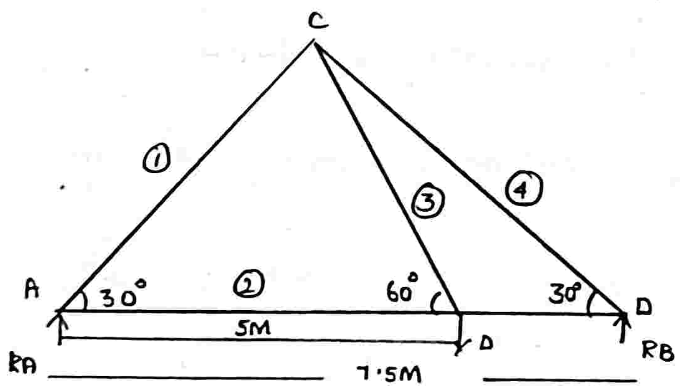
Soln:

let us first determine the reaction R_A and R_B

Taking moments about A, we get $R_B \times 7·5 = 5 \times 1$

$$R_B = \frac{5}{7·5} = \frac{2}{3}$$

$$= 0·667 kN$$



R_A = Total load − R_B

= 1 − 0.667

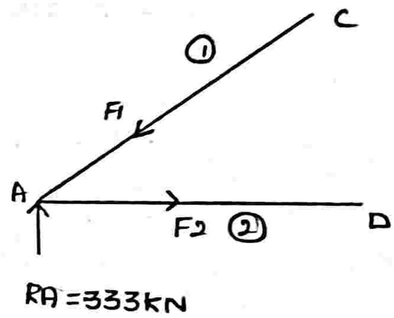
= 0.333 kN

Now consider the equilibrium of the various joints.

Joint A

F_1 = Force in member AC

F_2 = Force in member AD.



let the force F_1 is acting towards the joint A and F_2 is acting away from the joint A as shown in Fig

Resolving the forces vertically, we get

$$F_1 \sin 30^\circ = R_A$$

$$F_1 = \frac{R_A}{\sin 30^\circ} = \frac{0.333}{0.5}$$

= 0.666 kN (compressive)

Resolving the forces horizontally, we get

$$F_2 = F_1 \times \cos 30^\circ$$

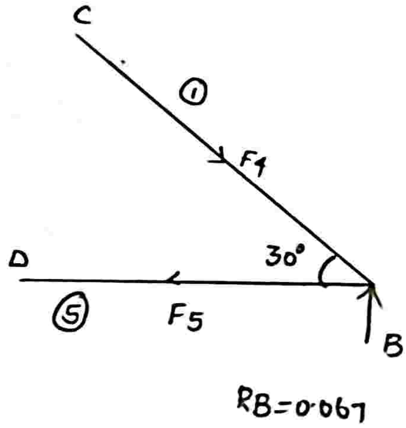
$$= 0.666 \times 0.866$$

$= 0.5767$ kN (Tensile)

Joint B

let F_4 = Force in member BC

F_5 = Force in member BD



let the direction of F_4 and F_5 are assumed as shown in Fig

Resolving the forces vertically, we get.

$F_4 \sin 30° = R_B = 0.667$

$$F_4 = \frac{0.667}{\sin 30°}$$

$= 1.334$ kN (compressive)

Resolving the force horizontally, we get.

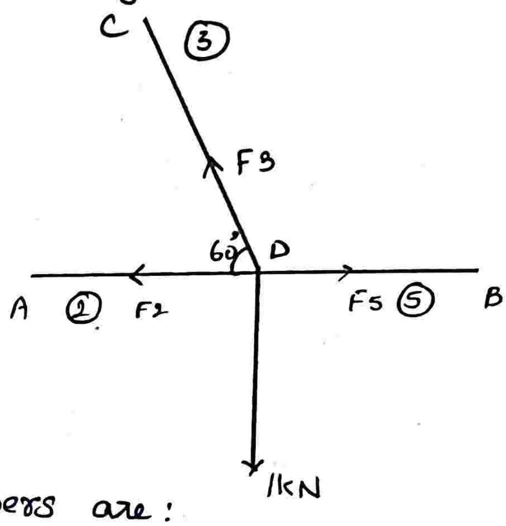
$F_5 = F_4 \cos 30° = 1.334 \times 0.866 = 1.155$ kN (Tensile)

Joint D.

let F_3 = Force in member CD. The forces F_2 and F_5 have been already calculated in magnitude and direction. The forces F_2 and F_5 are tensile and hence they will be pulling.

the joint D as shown in Fig. let the direction of F3 is assumed as shown in Fig.

Resolving the forces vertically, we get



$F_3 \sin 60^\circ = 1$

$$F_3 = \frac{1}{\sin 60^\circ} = \frac{1}{0.866}$$

$= 1.1547$ kN (Tensile)

Hence the forces in the members are:

$F_1 = 0.666$ kN (compressive)

$F_2 = 0.5767$ kN (Tensile)

$F_3 = 1.1547$ kN (Tensile)

$F_4 = 1.334$ kN (compressive)

$F_5 = 1.155$ kN (Tensile).

Problem 3:

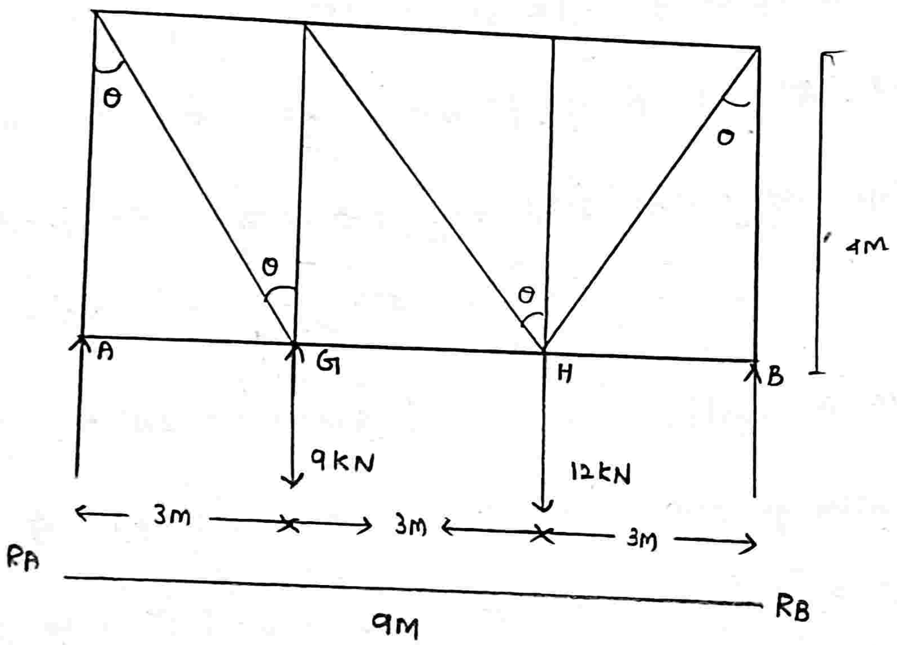
A truss of span 9m is loaded as shown in Fig 11.17: Find the reactions and forces in the member of the truss.

Soln :

Let us first calculate the reactions R_A and R_B

Taking moments about A, we get

$$R_B \times 9 = 9 \times 3 + 12 \times 6 \quad = 27 + 72 = 99$$



$$R_B = \frac{99}{9} \quad = 11 kN$$

$$R_A = \text{Total load} - R_B = (9+12) - 11 \quad = 10 kN.$$

In this problem, there are some members in which force is zero.

These members are obtained directly as given below.

If three forces act at a joint and two of them are along the same straight line, then for the equilibrium of the joint, the third force should be equal to zero."

1. Three forces are acting at the point A (i.e R_A, F_{AC} and F_{AG}), two of which (i.e R_A, R_{AC}) are along the same straight line. Hence the third force (ie, R_{AG}) is zero.

2. Similarly, three forces are acting at the joint B (ie R_B, F_{BF} and F_{BH}) two of which (ie, R_B and F_{BH}) are along the same straight line. Hence the third force F_{BH} should be zero.

3. At the joint E also, three forces (i.e, F_{ED}, F_{EF} and F_{EH}) are acting two of which (i.e F_{ED} and F_{EF}) are along the same straight line. Hence the third force F_{EH} must be zero.

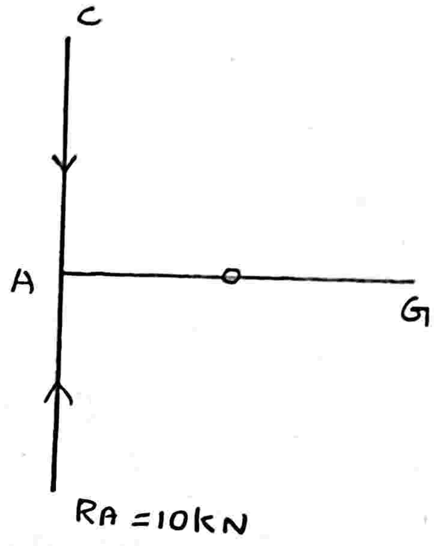
Now the equilibrium of various joints can be considered.

Joint A:

F_{AG} = Force in member AG = 0

F_{AC} = Force in member AC

= R_A = 10 kN (compressive)



Now consider the equilibrium of Joint C.

Joint C.

let F_{CD} = Force in Member CD

F_{CG} = Force in Member CG

F_{AC} = 10 kN (compressive).

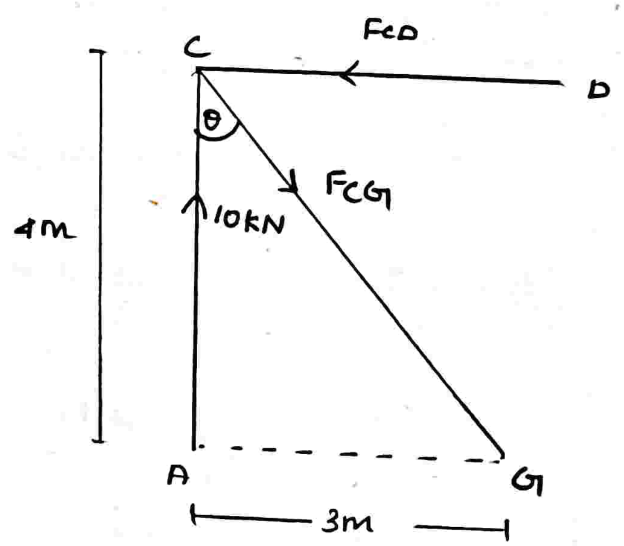
let the direction of F_{CG} and F_{CD} are assumed as shown in Fig

Resolving the forces vertically, we get

$$F_{CG} \cos\theta = 10$$

$$F_{CG} = \frac{10}{\cos\theta}$$

$$\cos\theta = \frac{AG}{CG} = \frac{4}{5}$$



$$F_{CG} = \frac{10}{(4/5)} = 10 \times \frac{5}{4} = 12.5\,kN \text{ (Tensile)}$$

Resolving forces horizontally, we get

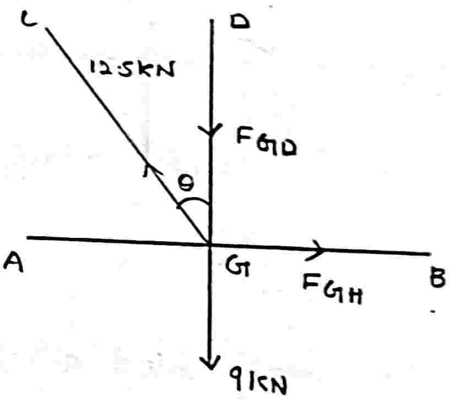
$$F_{CD} = F_{CG} \sin\theta$$

$$= 12.5 \times \frac{3}{5} = 7.5\,KN \text{ (compressive)}$$

Now consider the equilibrium of joint G.

Joint G.

The force in member CG is 12.5kN (Tensile).



Hence at the joint G, this force will be pulling the Joint G as shown in Fig 11.17 (c)

Resolving the forces vertically, we get

$$12.5 \cos\theta + F_{GD} = 9$$

$$F_{GD} = 9 - 12.5 \cos\theta$$

$$= 9 - 12.5 \times \frac{4}{5} \quad \left(\because \sin\theta = \frac{4}{5}\right)$$

$$= 9 - 10 = -1\,kN$$

As the magnitude of F_{GD} is negative, hence its assumed direction is wrong. The correct direction will be as shown in Fig.

Then

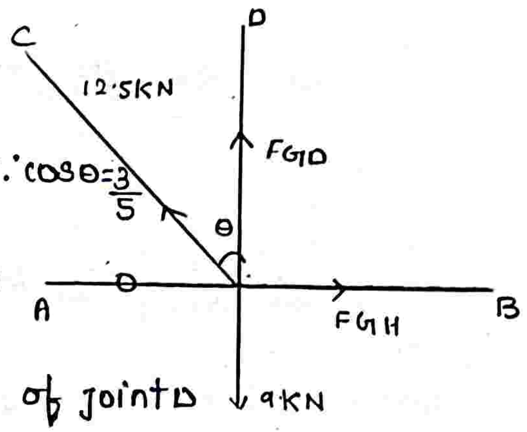
$$F_{GD} = 1KN \text{ (Compressive)}$$

Resolving the forces horizontally, we get

$$12.5 \sin\theta = F_{GH}$$

$$F_{GH} = 12.5 \times \frac{3}{5} \quad (\because \cos\theta = \frac{3}{5})$$

$$= 7.5KN \text{ (Tensile)}$$



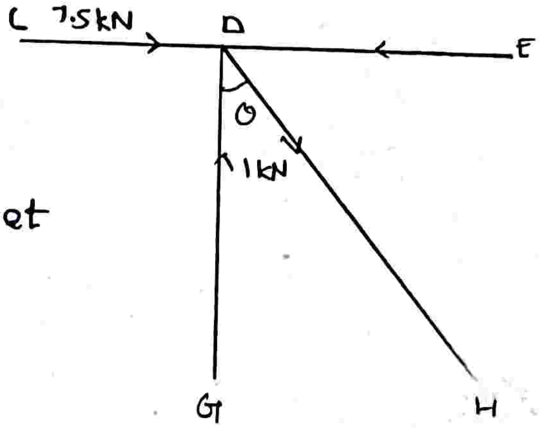
Now consider the equilibrium of joint D

Joint D:

The forces in the member CD and GD have been already calculated. They are 7.5 KN and 1 KN respectively. Both are compressive.

let F_{DH} = Force in member DH,

F_{DE} = Force in member DE



Resolving the forces vertically, we get

$$F_{DH} \cos\theta = 1KN$$

$$F_{DH} = \frac{1}{\cos\theta} = \frac{1}{(4/5)}$$

$= \frac{5}{4} = 1.25$ kN (Tensile)

Resolving the forces horizontally, we get

$$7.5 + F_{DH} \sin\theta = F_{DE} \qquad (\because \sin\theta = \frac{3}{5})$$

$$F_{DE} = 7.5 + 1.25 \times \frac{3}{5}$$

$$= 7.5 + 0.75 = 8.25 \text{ kN (compressive)}$$

Now consider the equilibrium of joint E

Joint E.

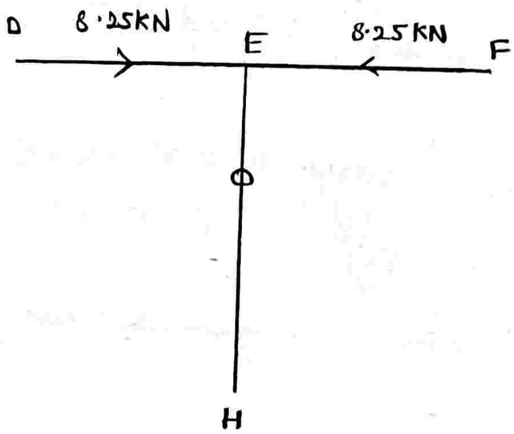
As shown in Fig. at joint E three force are acting. The force i.e F_{DE} and F_{EF} are in the same Straight Line.

Hence force F_{EH} Must be zero

Force in EF i.e, $F_{EF} = F_{DE}$

$= 8.25$ kN (Compressive)

Now consider the joint H.

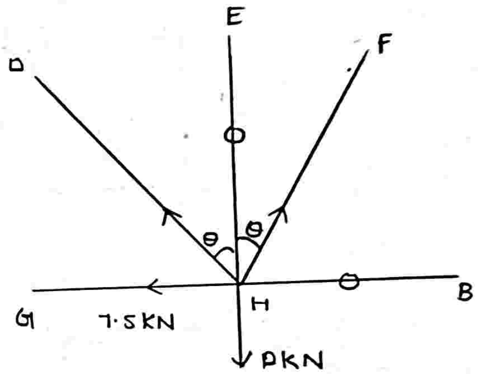


Joint H.

It is already shown that forces in the member EH and BH are zero.

Also the forces in the member GH is 7.5 KN tensile and in the member DH is 1.25 KN tensile.

let F_{HF} is the force in the member HF.



Resolving forces vertically, we get

$1.25 \cos\theta + F_{HF} \cos\theta = 12$

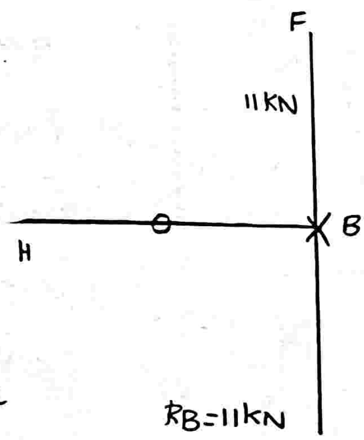
$1.25 \times \frac{4}{5} + F_{HF} \times \frac{4}{5} = 12$ $\left(\because \cos\theta = \frac{4}{5}\right)$

$1.0 + 0.8 F_{HF} = 12$

$F_{HF} = \frac{12 - 1.0}{0.8} = \frac{11}{0.8} = 13.75$ (Tensile)

Now consider the joint B.

Joint B

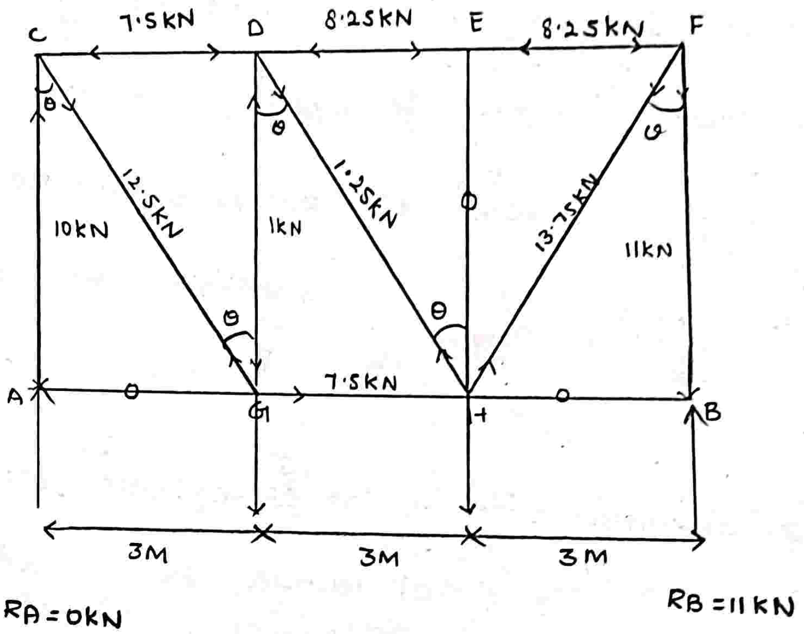


The force in member BF

=11KN (Compressive)

Now the forces in each Member are known.

They are shown in Fig. Also those forces are shown in a tabular form.



Member	Force in member.
AC	10 KN C (comp)
AG	0
CD	12.5 KN (Tens)
CG	7.5 KN (comp.)
DG	10 KN (comp.)
DE	8.25 KN (comp)
DH	1.25 KN (Tens.)
GH	7.5 KN (Tens.)
EH	0
EF	8.25 KN (comp.)
HB	0
HF	13.75 KN (Tens)

Method of Joints Applied to cantilever Trusses.

In case of cantilever trusses, it is not necessary to determine the support reactions. The Forces in the members of cantilever truss can be obtained by starting the calculations from the free end of the cantilever.

Problem 1:

Determine the forces in all the members of a cantilever truss shown in Fig.

Soln:

Here the calculations can be started from end c. Hence consider the equilibrium of the joint c.

Joint c.

let

F_{CD} = Force in member CD and

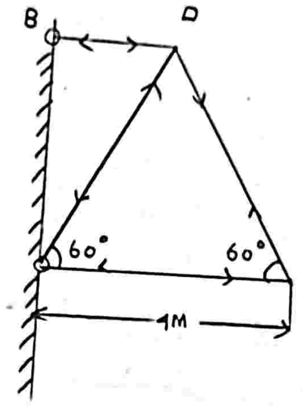
F_{CA} = Force in Member CA

Their assumed directions are shown in Fig

Resolving the force vertically, we get

$F_{CD} \times \sin 60^\circ = 1000$

$$F_{CD} = \frac{1000}{\sin 60^\circ} = \frac{1000}{0.866} = 1154.7N\ (T)$$



Resolving the forces horizontally, we get

$F_{CA} = F_{CD} \times \cos 60^\circ$

$= 1154.7 \times 0.5$

$= 577.35N$ (compressive)

Now consider the equilibrium of the Joint D.

Joint D

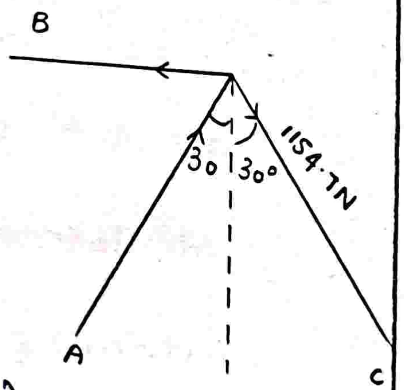
The force $F_{CD} = 1154.7N$ (tensile) is already calculated

F_{AD} = Force in member AD, and

F_{BD} = Force in member BD

Their assumed directions are shown in Fig.

Resolving the force vertically, we get.



$F_{AD} \cos 30^\circ = 1154.7 \cos 30^\circ$

$$F_{AD} = \frac{1154.7 \cos 30^\circ}{\cos 30^\circ} = 1154.7N$$

(comp.)

Resolving the force horizontally, we get

$$F_{BD} = F_{AD} \sin 30^\circ + F_{DC} \sin 30^\circ$$

$$= 1154.7 \times 0.5 + 1154.7 \times 0.5$$

$$= 1154.7 \text{ N (Tensile)}$$

Now the forces are shown in a tabular form below:

Member	Force in the member	Nature of force.
AC	577.35 N	compressive
CD	1154.7 N	Tensile
AD	1154.7 N	compressive
BD	1154.7 N	Tensile

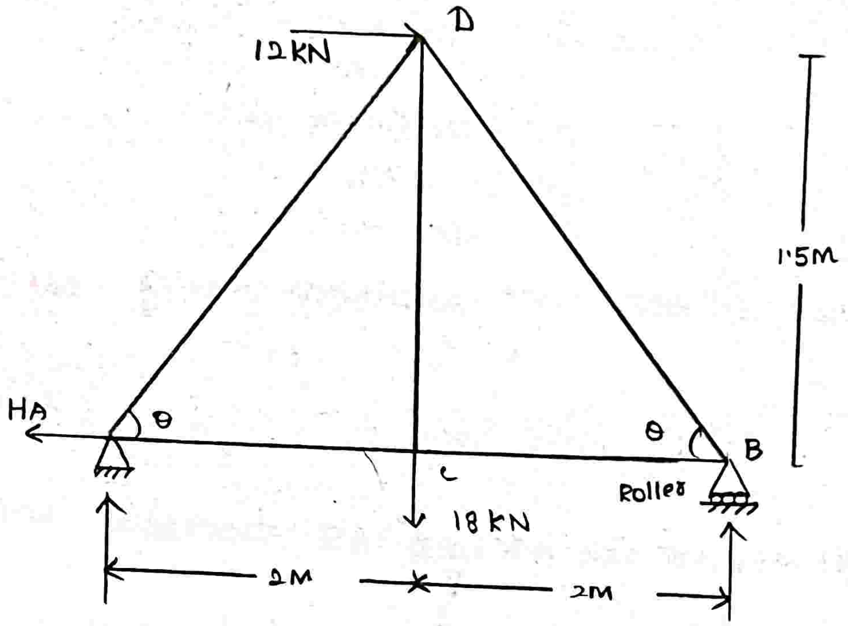
Problem 2:

Determine the force in the truss shown in Fig. Which carries a horizontal load of 12 kN and a vertical Load of 18 kN.

Soln:

The truss is supported on rollers at B and hence the reaction at B must be normal to the roller base i.e the reaction at B, in this case, should be vertical.

At the end A, the truss is hinged and hence the support reaction at the hinged end A will consists of a horizontal reaction HA and a vertical reaction RA.



Taking moment of all forces at A, we get

$$R_B \times 4 = 18 \times 2 + 12 \times 1.5 = 36 + 18 = 54$$

$$R_B = \frac{54}{4} = 13.5\ KN\ (\uparrow)$$

$$R_A = \text{Total vertical load} - R_B$$

$$= 18 - 13.5 = 4.5\ KN\ (\uparrow)$$

$$H_A = \text{Sum of all horizontal loads}$$

$$= 12\ KN\ (\leftarrow)$$

Now the force in the members can be calculated.

In triangle BCD, $BD = \sqrt{BC^2 + CD^2}$

$= \sqrt{2^2 + 1.5^2}$

$= 2.5 m$

$\cos\theta = \frac{BC}{BD} = \frac{2}{2.5} = 0.8$

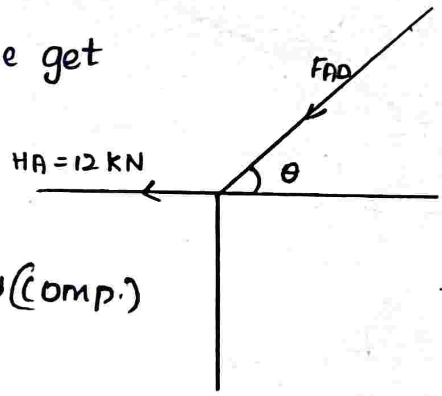
$\sin\theta = \frac{CD}{BD} = \frac{15}{2.5} = 0.6$

let us first consider the equilibrium of joint A.

Joint A.

The reactions RA and HA are known in magnitude and direction. Let the directions of the forces in the members AC and AD are as shown in Fig

Resolving the force vertically, we get



$F_{AD} \sin\theta = R_A$

$F_{AD} = \frac{R_A}{\sin\theta} = \frac{4.5}{0.6} = 7.5 kN$ (Comp.)

or

Resolving the forces horizontally, we get

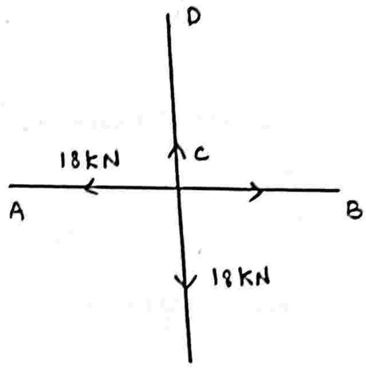
$F_{AC} = H_A + F_{AD} \cos\theta$

$= 12 + 7.5 \times 0.8 = 18 kN$ (Tens)

Now consider the joint c.

Joint c.

At the joint c, the force D in member CA and vertical Load 18 KN are known in magnitude and directions. For equilibrium of the joint c.



$F_{BC} = F_{CA} = 18\,KN$ (Tensile)

$F_{CD} = 18\,KN$ (Tensile)

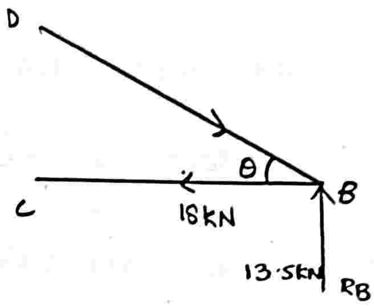
Now consider the joint B.

Joint B.

At the joint B, RB and Force FBc are known in magnitude and direction.

Let FBD is the force in member BD

Resolving the forces vertically, we get



$F_{BD} \times \sin\theta = R_B$

$$F_{BD} = \frac{R_B}{\sin\theta} = \frac{13.5}{0.6}$$

$= 22.5\,KN$ (comp.)

Now the forces are shown in tabular form below.

Member	Force in member	Nature of force
AC	18KN	Tensile
AD	7·5KN	compressive
CD	18 KN	Tensile
CB	18 KN	Tensile
BD	2·5KN	Compressive.

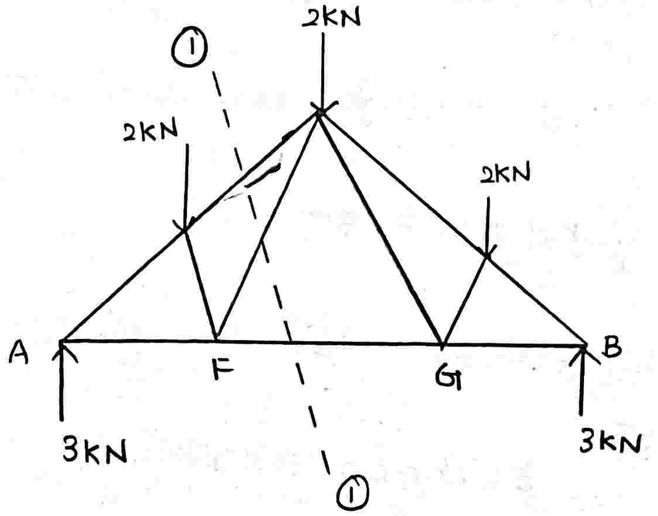
Method of Section:

When the force in a few members of a truss are to be determined, then the method of Section is mostly used. This method is very quick as it does not involve the solution of other joints of the truss.

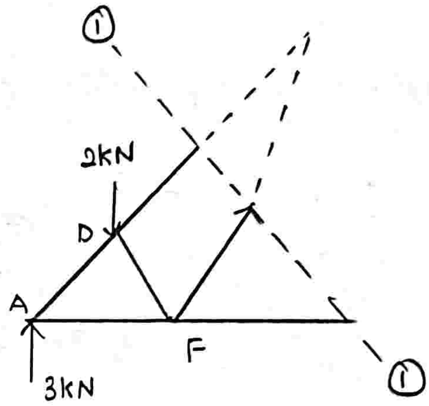
In this method, a section line is passed through the members, in which forces are to be determined as show in fig. The section Line should be drawn in such a way that it does not cut more than three members in which the forces are unkown. The part of the truss, on any one side of the Section Line, is treated as a free body in equilibrium under the action of external forces on the part and.

Forces in the members cut by the section line. The unknown force in the members are then determined by using equations of equilibrium as.

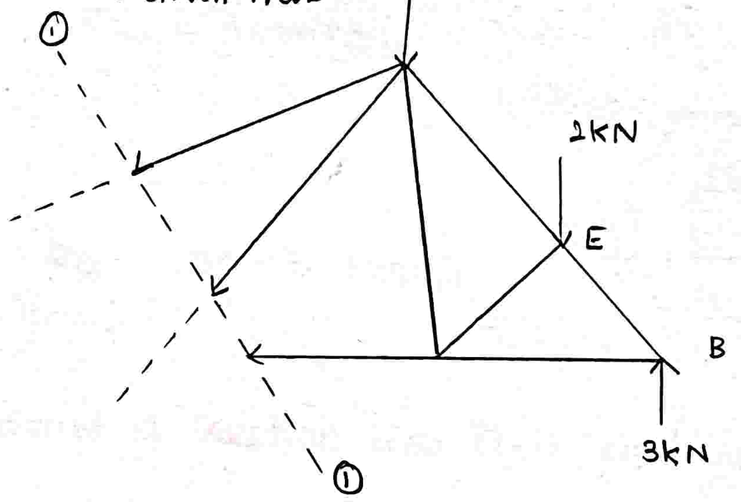
$$\Sigma F_x = 0 , \; \Sigma F_y = 0 \text{ and } \Sigma M = 0$$



a) Given Truss



b) left part



c) Right part

If the magnitude of the forces, in the members cut by a section line, is postive then the assumed direction is correct. If magnitude of a force is negative, then reverse the direction of that force.

Problem 1:

Find the forces in the members AB and AC of the truss shown in Fig. using method of section.

Sol:

First determine the reaction R_B and R_C.

The distance of line of action of 20kN from point B is

$$AB \times \cos 60^\circ \text{ or } 2.5 \times \frac{1}{2} = 1.25\,M$$

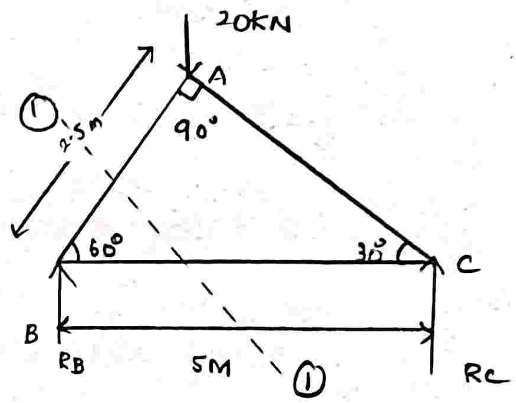
Taking moments about point B, we get

$$R_C \times 5 = 20 \times 1.25$$

$$R_C = \frac{20 \times 1.25}{5}$$

$$= 5\,KN$$

$$R_B = 20 - 5 = 15\,KN$$



Now draw a section line (1-1), cutting the members AB and BC in which forces are to be determined.

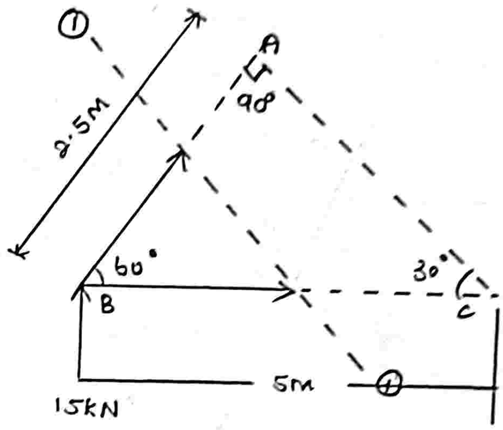
Now consider the equilibrium of the left part of the truss. This part is shown in Fig.

let the directions of F_{BA} and F_{BC} are assumed as shown in Fig.

Now taking the moments of all the forces acting on the left part about point c, we get

$$15 \times 5 + (F_{BA} \times AC)^{*} = 0$$

(∵ The perpendicular distance b/w the Line of action of FBA and point c is equal to Ac)



$$75 + F_{BA} \times 5 \times \cos 30^\circ = 0$$

$$F_{BA} = \frac{-75}{5 \times \cos 30^\circ} = -17.32\,kN$$

The negative sign shows that FBA is acting in the opposite direction (ie., towards point B).

Hence Force FBA will be a compressive force.

$$F_{BA} = 17.32\ kN\ (\text{compressive}).$$

Again taking the moments of all forces acting on the left part about point A, we get 15x prependicular distance between the Line of action of

$$15\,KN \times 2.5 \times \cos 60^\circ = F_{BC} \times 2.5 \times \sin 60^\circ$$

$$F_{BC} = \frac{15 \times 2.5 \times \cos 60^\circ}{2.5 \times \sin 60^\circ} = \frac{15 \times 0.5}{0.866}$$

$= 8.66 kN$ (Tensile).

Problem 2.

A truss of span 5 m is loaded as shown in Fig. Find the reaction and forces in the members marked 4, 5 and 7 using method of section.

Soln:

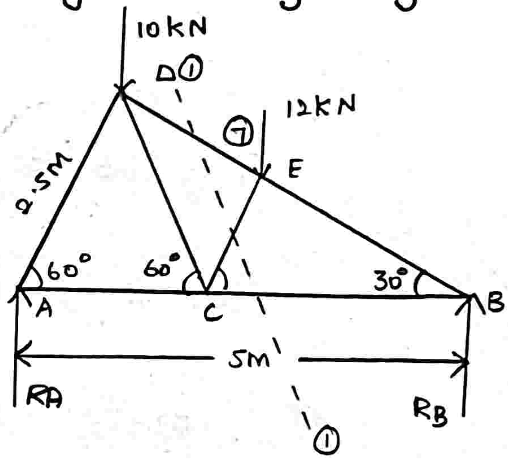
let us first determine the reactions R_A and R_B

Triangle ABD is a right angled triangle having angle

$ADB = 90°$

$AD = AB \cos 60°$

$= 5 \times 0.5 = 2.5 M$



The distance of line of action the vertical load 10kN from point A will be AD cos 60° or $2.5 \times 0.5 = 1.25 M$

From triangle ACD, we have

$AC = AD = 2.5 M$

$BC = 5 - 2.5 = 2.5 M$

In right angled triangle CEB, we have

$BE = BC \cos 30° = 2.5 \times \frac{\sqrt{3}}{2}$

∴ The distance of line of action of vertical load 12KN from point B will be BE cos 30° or BE $\times \frac{\sqrt{3}}{2}$

$$= \left[2.5 \times \frac{\sqrt{3}}{2}\right] \times \frac{\sqrt{3}}{2} = 1.875\,m$$

∴ The distance of the line of action of the load of 12KN from point A will be

$$(5 - 1.875) = 3.125\,M$$

Now taking the moments about A, we get

$$R_B \times 5 = 10 \times 1.25 + 12 \times 3.125 = 50$$

$$R_B = \frac{50}{5} = 10KN \text{ and } R_A = (10+12) - 10$$
$$= 12KN$$

Now draw a section line (1-1); cutting the member 4, 5 and 7 in which forces are to be determined. Consider the equilibrium of the right part of the truss (because it is smaller than the left part).

This part is shown in Fig. Let F_4, F_5 and F_7 are the forces in member 4, 5 and 7. Let their directions are assumed as show in Fig

Now taking the moments of all the forces acting on the right part about point E, we get

$$R_B \times BE \cos 30^\circ = F_4 \times (BE \times \sin 30^\circ)$$

$$10 \times \left(2.5 \times \frac{\sqrt{3}}{2}\right) \times \frac{\sqrt{3}}{2} = F_4 \times 2.5 \times \frac{\sqrt{3}}{2} \times 0.5$$

$$10 \times \frac{\sqrt{3}}{2} = F_4 \times 0.5$$

$$F_4 = 10 \times \frac{\sqrt{3}}{2} \times \frac{1}{0.5}$$

$$= 17.32 \text{ kN (Tensile)}$$

Now taking the moments of all the forces about point B acting on the right part, we get

$$12 \times BE \cos 30^\circ + F_5 \times BE = 0$$

$$12 \times \cos 30^\circ + F_5 = 0$$

$$F_5 = -12 \times \cos 30^\circ$$

$$= -10.392 \text{ kN}.$$

-ve sign indicates that F5 is compressive

F5 = 10·392 kN (compressive)

Now taking the moment about point c of all the force acting on the right parts, we get

$$12 \times (2.5 - BE \cos 30^\circ) = F7 \times CE + RB \times BC$$

$$12 \times \left[2.5 - 2.5 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right] = F7 \times 2.5 \times \sin 30^\circ + 10 \times 2.5$$

$$12 \times (2.5 - 1.875) = F7 \times 1.25 + 25 \text{ or } 7.5$$

$$= 1.25 F7 + 25$$

$$F7 = \frac{7.5 - 25}{1.25}$$

$$= -14 \text{ kN}.$$

Negative sign shows that F7 is compressive

F7 = 14 kN (compressive).

These forces are same as obtained in problem.

problem 3.

A truss of span 9m is loaded as shown in Fig. Find the reaction and force in the members marked 1, 2 and 3.

Soln:

Let us first calculated the reactions R_A and R_B

Taking moment abouts A, we get

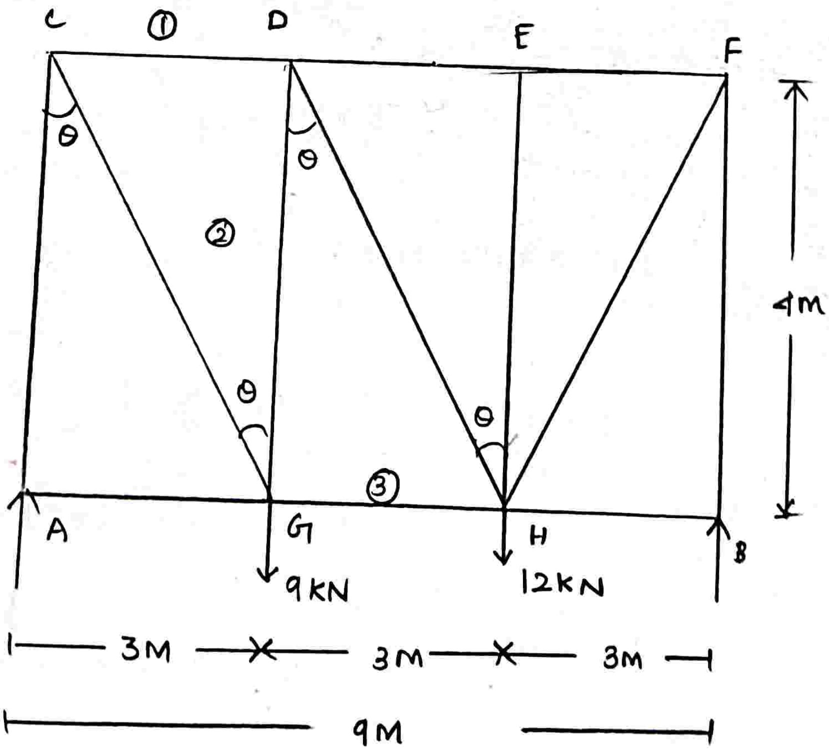
$$R_B \times 9 = 9 \times 3 + 12 \times 6 = 27 + 72 = 99$$

$$R_B = \frac{99}{2}$$

$$= 11 \text{ kN}$$

$$R_A = (9+12) - 11$$

$$= 10 \text{ kN}.$$



Now draw a section line (1-1). cutting the members 1, 2 and 3 in which forces are to be determined. consider the equilibrium of the part of the truss (because it is smaller than the right part). This part is shown in Fig. Let F_1, F_2 and F_3 are the force member 1, 2 and 3 respectively. Let their directions are assumed as shown in Fig.

Taking moments of all the forces acting on the left part about point D, we get.

$$10 \times 3 = F_3 \times 4$$

$$F_3 = \frac{10 \times 3}{4}$$

$$= 7.5 \text{ KN} \quad \text{(Tensile)}$$

Now taking the moments of all the forces acting on the left part about point G, we get

$$10 \times 3 + F_1 \times 4 = 0$$

$$F_1 = -\frac{30}{4} = -7.5 \text{ KN}$$

Negative sign shows that force F_1 is compressive.

$F_1 = 7.5$ kN (compressive).

Now taking the moments about the point c, we get

$$F_2 \times 3 - 9 \times 3 + F_3 \times 4 = 0$$

$$F_2 \times 3 - 27 + 7.5 \times 4 = 0$$

$$F_2 = \frac{27 - 7.5 \times 4}{3}$$

$$= \frac{-3}{3}$$

$$= -1.0 \text{ kN}$$

Negative sign shows that force F_2 is compressive.

$F_2 = 1.0$ kN (compressive).